A Cut Approach for Solving the Single-Row Machine Layout Problem

(Society of Industrial and Applied Mathematics, 2014 SIAM Annual Meeting)
Session: CP30 Optimization: Friday, July 11, 4:00 PM - 6:00 PM

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Abstract

A cut approach is developed for the machine configuration in a single-row linear layout, with objective to minimize the total material movement cost. We show that the above equidistant linear layout problem is equivalent to the locale network problem by graph theory. The minimum cut, which has the minimum capacity in each locale network, is the corresponding optimal layout that minimizes the material handling cost. The computational complexity of the cut approach is $O(n^3)$, where $n$ is the number of workstations in the layout problem. In addition, the non-equidistant linear layout problem can be transformed into an equidistant linear layout problem; and then solved with the extended approach.

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Keywords: Facility layout, linear flow-line, network flow, cut approach

Approximately word counts: 4000
1. Introduction

In the flexible manufacturing environment, particularly, flow lines or linear row assembly system, workstations or machines are arranged along a straight (or U, S-shaped) line where a material handling system moves parts from one location to another. A popular implementation of such linear layouts can be found for the assembly of 3-C products such as: cell phone, tablet, and notebook computer. These layouts have been widely used in industry, due to the flexibility to use a variety of material handling devices such as conveyor, towline, crane, and wired path of automated guided vehicles. The most common configuration in a linear layout can be described as follows. An input station is located at one end (left) of the linear layout and an output station is located at another end (right). The part or material enters the system from the input station, move along the linear path to the designated workstation for processing or assembly, and finally leaves the system from the output station. The material movement from left to right is called forward movement. Otherwise, it is termed as backtrack movement when the material handling device moves in the reversed direction, based on the routing requirement.

The most frequently described formulation for modeling the facility layout problem is the well-known Quadratic Assignment problem (QAP), see Koopmans & Beckman (1957), which is also known to be \(NP\)-complete (Sahni and Gonzalez, 1976). Both heuristics and branch and bound methods have been described to solve these difficult problems, e.g., see Lawler (1963), and Bazaraa and Kirca (1983). However, different layout approaches are required for the specific material flow features in the manufacturing system, see Kusiak and Heragu (1987). These manufacturing layouts include linear, single and double row, U-shape, circular, unicycle, and tandem configuration. Heragu and Kusiak (1988) also discussed various machine layout problems in manufacturing systems. Kaku and Rachamadugu (1992) considered the loop conveyor and linear track conveyor layout problems. Hassan (1995) described the layout design in group technology manufacturing systems. Both layout models are based on the classical QAP, and heuristics are proposed to solve these problems.

The existing optimization problem closest to the described linear layout that has been discussed is the Generalized Linear Ordering Problem (GLOP), see Karp & Held (1967), Adolphson & Hu (1973), and Picard & Quwyranne (1981). GLOP can be treated as a one-dimensional version of the QAP, where all machines are to be assigned to the layout so that the total movement cost (in both directions) among machines is minimized, see Sanjeevi & Kianfar (2010) for a recent study. It has been shown to be \(NP\)-complete. The main difference
of the linear row layout is that the input and output stations in a linear layout problem are fixed at both ends, and machines are to be located in between. Existing approaches for solving the linear layout problem have focused on heuristics to solve more realistic problems with 35 machines; and exact solution procedures are used for solving small problems with 8 or 10 machines, based on the complexity analysis of GLOP. Kouvelis et al. (1995), and Kouvelis & Chang (1996) described both dynamic programming and heuristics when the machines are to be placed at equal distance and total backtracking movement cost is to be minimized. A branch and bound approach, augmented with Tabu search is reported in Palubeckis (2012), where the solvable problem size is increased to 35 machines. More recent heuristics can be found in Kumar et al. (1995), Sarker et al. (1995), Sarker et al. (1998), Yu & Sarker (2003), Datta et al. (2011).

The findings of this paper are two folds. The linear row layout problem can be successfully transformed into a network flow problem, and the optimum layout can be found by solving a sequence of minimum cut problems in the constructed graph. An efficient cut approach that runs at the cubic order of the number of workstations is then developed to solve the layout problem. In the next section, we first describe the linear row layout problem for the bi-directional material flow system. Optimality conditions using the locale network are then described to characterize the optimum layout. Section 3 gives the polynomial time solution procedure to solve the linear layout problem; and an example is given to illustrate the solution approach. The last section provides a brief summary.

2. Problem Description and Network Transformation

We consider the manufacturing layout problem in which a bi-directional material flow system is used for the material handling. The manufacturing system is used to process a group of parts that route through a given sequence of workstations. Each part has a unique routing sequence, which is assumed fixed and known. The workstations are to be located along the linear path with equal distance, so as to minimize the total material flow cost. Linear row layouts are extensively used in many automated manufacturing systems, due to the wide use of efficient material handling networks. Such a network connects all workstations by a path passing through each workstation exactly once. This material handling network may represent a conveyor, towline, overhead monorail system, or guided-path automated guided vehicles. The common operational strategy requires that all part to enter from the input station and exit the system at the output station. Therefore, all parts pass
through all stations along the network before leaving the system. When one part completes its processing at a particular station, it proceeds immediately to the next workstation by moving along the linear material handling network, until the entire routing of this part is completed. In its simplest form, the movement of a part from one workstation to another is equivalent to that the material handling device departs from the input station, moves to the workstation to pick up the part, delivers it to another workstation, and finally moves to the output station. A forward movement takes place when the part routes from one workstation to another where the first workstation is located at upstream and another is located at downstream of the linear network. When these workstations are located in reversed direction, backtracking movement is required.

Let \( i = 0, 1, \ldots, n + 1 \) denote the number of locations for the linear layout, where \( i = 0 \) denotes the input station, \( i = n + 1 \) denotes the output station, and \( L \) is the minimal distance between any two locations. Also let \( t_{ij} \) be the amount of flow from workstation \( i \) that must be routed directly to workstation \( j \), for \( 0 \leq i, j \leq n + 1 \). Let \( \alpha = \{\alpha(1), \ldots \alpha(i), \ldots \alpha(j), \ldots \alpha(n)\} \) be the location assignment vector for the \( n \) workstations. In the linear network, if \( \alpha(i) < \alpha(j) \), workstation \( i \) located at upstream and workstation \( j \) is located at downstream of the network. Thus, \( t_{ij} \) is the amount of forward flow from workstation \( i \) to \( j \), while \( t_{ji} \) is the amount of backward flow for these two workstations, and \( t_{ii} = 0 \) for all \( i = 1, \ldots n \). Figure 1 below depicts a typical linear row layout.

For a given location vector \( \alpha \) of the \( n \) workstations, the objective function to be minimized is:

\[
F(\alpha) = \sum_{i=0}^{n+1} \sum_{j=0}^{n+1} t_{ij} |\alpha(i) - \alpha(j)|. \tag{1}
\]
And our objective is to find a permutation sequence \( \alpha \) of workstations so that \( F(\alpha) \) is minimized. Let \( S_q = \{ j \mid \alpha(j) \leq q \} \) be the set of workstations that are located before the \( q^{th} \) sequence in the linear layout, \( \overline{S}_q = \{ j \mid \alpha(j) > q \} \) be the set of workstations after that, and \( C_q(S_q, \overline{S}_q) \) is the cut value (total material flow cost) for the corresponding network, i.e., q-locale network, see Picard & Ratliff (1978), then:

\[
C_q(S_q, \overline{S}_q) = \sum_{i \in S_q} t_{0i} + \sum_{i \in \overline{S}_q} t_{in} + \sum_{i \in S_q} \sum_{j \in \overline{S}_q} t_{ij}.
\]  

(2)

For a layout vector \( \alpha \), the total material flow cost can be rewritten as below:

\[
F(\alpha) = \sum_{i=1}^{n} t_{0i} + \sum_{i=1}^{n} t_{i,n+1} + \sum_{q=1}^{n-1} C_q(S_q, \overline{S}_q).
\]  

(3)

The equivalence of the above two problems can be illustrated in the Figure 2 below.

![Figure 2. The equivalent transformation](image)

Proposition 1: Let \( (S'_q, \overline{S}'_q) \) be one of the minimal cut from the q-locale network and \( \alpha' \) be the corresponding location vector, there exists a minimum cut \( C_q(S_q, \overline{S}_q) \) and layout sequence \( \alpha^* \) such that:
\[ \alpha'(j) = \alpha'(j) - 1, \quad \text{if} \quad j \in S_q' \cap \overline{S}_q \]
\[ \alpha'(j) = \alpha'(j) + 1, \quad \text{if} \quad j \in S_q' \cap S_q^* \]
\[ \alpha'(j) = \alpha'(j) \quad \text{otherwise}, \]

for any workstation \( j, 1 \leq j \leq n \), and \( F(\alpha^*) - F(\alpha') \leq C_q(S_q', \overline{S}_q^*) - C_q(S_q, \overline{S}_{q^*}) \).

Proof: The difference of objective values of the above two location vectors can be rewritten as:
\[
F(\alpha^*) - F(\alpha') = \sum_{j \in S_q' \cap S_q} (t_{of} - t_{j, n+1}) - \sum_{j \in S_q' \cap \overline{S}_q} (t_{of} - t_{j, n+1}) \\
+ \sum_{j \in S_q' \cap \overline{S}_q} \sum_{k \in S_q' \cap \overline{S}_q} t_{jk} \left\{ [\alpha'(j) - [\alpha'(k) - 1]] - [\alpha'(j) - \alpha'(k)] \right\} \\
+ \sum_{j \in S_q' \cap \overline{S}_q} \sum_{k \in \overline{S}_q' \cap \overline{S}_q} t_{jk} \left\{ [\alpha'(j) - [\alpha'(k) + 1]] - [\alpha'(j) - \alpha'(k)] \right\} \\
+ \sum_{j \in \overline{S}_q' \cap \overline{S}_q} \sum_{k \in S_q' \cap \overline{S}_q} t_{jk} \left\{ [\alpha'(j) - [\alpha'(k) - 1]] - [\alpha'(j) - \alpha'(k)] \right\} \\
+ \sum_{j \in \overline{S}_q' \cap \overline{S}_q} \sum_{k \in \overline{S}_q' \cap \overline{S}_q} t_{jk} \left\{ [\alpha'(j) - [\alpha'(k) + 1]] - [\alpha'(j) - \alpha'(k)] \right\}.
\]

Based on definition of the minimal cut in the q-locale network, the above equation can be expressed as
\[
C_q(S_q, \overline{S}_q^*) - C_q(S_q, \overline{S}_q) = \sum_{j \in S_q' \cap S_q} (t_{j, n+1} - t_{0j}) + \sum_{j \in S_q' \cap \overline{S}_q} (t_{0j} - t_{j, n+1}) \\
- \sum_{j \in S_q' \cap S_q} \sum_{k \in S_q' \cap \overline{S}_q} t_{jk} + \sum_{j \in S_q' \cap \overline{S}_q} \sum_{k \in \overline{S}_q' \cap \overline{S}_q} t_{jk} \\
+ \sum_{j \in \overline{S}_q' \cap \overline{S}_q} \sum_{k \in S_q' \cap \overline{S}_q} t_{jk} - \sum_{j \in \overline{S}_q' \cap \overline{S}_q} \sum_{k \in \overline{S}_q' \cap \overline{S}_q} t_{jk}.
\]

A term-by-term comparison indicates that: \( F(\alpha^*) - F(\alpha') \leq C_q(S_q', \overline{S}_q^*) - C_q(S_q, \overline{S}_q) \).

Proposition 2: There exists a set of q-locale networks with the minimum cut \( C_q(S_q, \overline{S}_q) \),
\[ S_q = \{ j \mid \alpha^*(j) \leq q \} \quad \text{and} \quad \overline{S}_q = \{ j \mid \alpha^*(j) > q \} \], for \( q = 1, \ldots, n - 1 \), where \( \alpha^* \) is the optimum layout sequence with the minimum total flow cost, and vice versa.

Proof: From Eq. (2), it is easy to see that the objective value of layout vector \( \alpha^* \) consists of three parts: flow cost from input station to each workstation, flow cost from each workstation to the output station, and flow cost between workstations. From Eq. (3), the minimum cut for the q-locale network corresponds to the minimum flow cost for the layout vector at this location. Hence, the set of minimum cuts of the q-locale network, for \( q = 1, \ldots, n - 1 \),
corresponds to the optimum layout. Conversely, assumed that the cut corresponding to \( \left( S'_q, \overline{S'_q} \right) \) is not the minimum. From the preceding proposition, we can find a minimum cut that corresponds to \( C_q \left( S_q, \overline{S_q} \right) \) of the q-locale network, for \( q = 1, \ldots, n-1 \), with a reduced cut value, since \( C_q \left( S'_q, \overline{S'_q} \right) - C_q \left( S_q, \overline{S_q} \right) \geq 0 \). This also implies that \( F(\alpha') - F(\alpha) \leq 0 \), which shows that \( \alpha^* \) must be the optimum layout sequence.

Note that the \( q = n \) locale network need not be solved, since there is only one unassigned workstation with one open location at this stage. The preceding properties can be used to devise an efficient solution procedure to find the optimal layout sequence, which is described in the next section.

3. Solution Procedure and illustration

We first propose an efficient algorithm to find the optimal layout sequence. A numerical example is used to illustrate the described approach. During the iterative solution process, let \( t_{ij} \) be the amount of flow from assigned workstation \( i \) (which has been assigned to the location, 1, \ldots, \( q \)) to the unassigned workstation \( j \) (which has not been assigned to the location, 1, \ldots, or \( q \)), and \( w_{ij} \) be the amount of flow from unassigned workstation \( i \) to the unassigned workstation \( j \), \( 0 \leq i, j \leq n+1 \), for the q-locale network, \( q = 1, \ldots, n-1 \). At the first iteration, only the input station and output station are assigned workstations, i.e., the input station is assigned to the location 0, and the output station is assigned to the last location \( n+1 \). All other workstations are designated as unassigned.

{The cut approach for linear row layout}
1. Let \( S_q = \{0\} \), i.e., it includes only the input station.
2. For \( q = 1 \), Find the minimum cut for the 1-locale network as follows.
   1) Generate a network (graph) using the input station as source node and the output station as sink node. All \( n \) workstations are placed in between, i.e., every workstation is a candidate to be assigned to the first location. The arc capacity for any two nodes \( i \) and \( j \) is the part flow in between.
   2) Compute \( n \) cut values: \( C_q \left( S_q, \overline{S_q} \right) = \sum_{i \in S_q} t_{oi} + \sum_{i \in S_q} t_{i,n+1} + \sum_{i \in S_q} \sum_{j \in \overline{S_q}} w_{ij} \). Find the minimum cut.
3) Assign workstation $i^*$ in this minimum cut to the $q^{th}$ layout sequence. Tie breaking is arbitrary.

3. Update $S_q = S_q \cup \{i^*\}$. Increase $q$ by 1 and repeat steps 2) and 3) until $q = n-1$. Go to the next step

4. The optimum layout sequence $\alpha^*$ has been found; and the total minimum flow cost is

$$F(\alpha) = \sum_{i=1}^{n} t_{0i} + \sum_{i=1}^{n} t_{i,n+1} + \sum_{q=1}^{n-1} C_q(S_q, \bar{S_q})$$

Note that the largest network problem is generated at 1-locale network, where $n$ workstations are all unassigned. In this case, there are $n$ possible cut combinations, and each one requires $(2n-1)$ computations to find the minimum cut. The number of nodes (the number of unassigned workstations) is decreased by one at each iteration. Thus, to find the optimum layout, the total computations for the $q$-locale networks are:

$$\sum_{i=1}^{n} i \cdot (2i - 1) = 2 \sum_{i=1}^{n} i^2 - \sum_{i=1}^{n} i = \left[ \frac{n \cdot (n+1) \cdot (2n+1)}{6} \right] - \left[ \frac{n \cdot (n+1)}{2} \right].$$

That is, the computational complexity of this cut approach is $O(n^3)$, a cubic function for the problem size $n$.

As an illustration, we consider a linear flow layout problem with 6 workstations, where the input station (0) is assigned to location 0, and the output station (7) is assigned to location 7. The amount of part flow from one workstation to all the remaining stations is given in the Table 1 below:

<table>
<thead>
<tr>
<th>$t_{ij}$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>--</td>
<td>20</td>
<td>5</td>
<td>0</td>
<td>12</td>
<td>0</td>
<td>14</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>--</td>
<td>24</td>
<td>12</td>
<td>0</td>
<td>15</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>0</td>
<td>--</td>
<td>10</td>
<td>0</td>
<td>0</td>
<td>15</td>
<td>14</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>19</td>
<td>0</td>
<td>--</td>
<td>0</td>
<td>12</td>
<td>0</td>
<td>10</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>12</td>
<td>10</td>
<td>14</td>
<td>--</td>
<td>0</td>
<td>0</td>
<td>15</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>29</td>
<td>--</td>
<td>10</td>
<td>12</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>5</td>
<td>10</td>
<td>24</td>
<td>--</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>--</td>
<td>--</td>
</tr>
</tbody>
</table>

Based on the above flow matrix, the flow matrices for the q-locale network are as follows:
Table 2. The flow matrix $T(i_j)$ for the q-locale network

<table>
<thead>
<tr>
<th>$i$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>20</td>
<td>5</td>
<td>0</td>
<td>12</td>
<td>0</td>
<td>14</td>
</tr>
<tr>
<td>7</td>
<td>0</td>
<td>14</td>
<td>10</td>
<td>15</td>
<td>12</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 3. The flow matrix $W(i_j)$ for the q-locale network

<table>
<thead>
<tr>
<th>$i$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>--</td>
<td>24</td>
<td>31</td>
<td>12</td>
<td>15</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>24</td>
<td>--</td>
<td>10</td>
<td>10</td>
<td>0</td>
<td>15</td>
</tr>
<tr>
<td>3</td>
<td>31</td>
<td>10</td>
<td>--</td>
<td>14</td>
<td>12</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>12</td>
<td>10</td>
<td>14</td>
<td>--</td>
<td>29</td>
<td>10</td>
</tr>
<tr>
<td>5</td>
<td>15</td>
<td>0</td>
<td>12</td>
<td>29</td>
<td>--</td>
<td>34</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
<td>15</td>
<td>5</td>
<td>10</td>
<td>34</td>
<td>--</td>
</tr>
</tbody>
</table>

For $q = 1$, i.e., the 1-locale network, the computational details are listed below:

Table 4. The computational details of the cut approach for the 1-locale network

$q = 1$  
| $S_1 = \{1,\cdots\}$ | $C_1(S_1, \overline{S}_1) = (t_{02} + t_{03} + t_{04} + t_{05} + t_{06}) + t_{17}$  
+ $(w_{12} + w_{13} + w_{14} + w_{15} + w_{16}) = 113$  
$\overline{S}_1 = \{2, 3, 4, 5, 6\}$ |
| $S_1 = \{2,\cdots\}$ | $C_1(S_1, \overline{S}_1) = (t_{01} + t_{03} + t_{04} + t_{05} + t_{06}) + t_{27}$  
+ $(w_{12} + w_{23} + w_{24} + w_{25} + w_{26}) = 119$  
$\overline{S}_1 = \{1, 3, 4, 5, 6\}$ |
| $S_1 = \{3,\cdots\}$ | $C_1(S_1, \overline{S}_1) = (t_{01} + t_{02} + t_{04} + t_{05} + t_{06}) + t_{37}$  
+ $(w_{13} + w_{23} + w_{34} + w_{35} + w_{36}) = 133$  
$\overline{S}_1 = \{1, 2, 4, 5, 6\}$ |
| $S_1 = \{4,\cdots\}$ | $C_1(S_1, \overline{S}_1) = (t_{01} + t_{02} + t_{03} + t_{05} + t_{06}) + t_{47}$  
+ $(w_{14} + w_{24} + w_{34} + w_{45} + w_{46}) = 129$  
$\overline{S}_1 = \{1, 2, 3, 5, 6\}$ |
| $S_1 = \{5,\cdots\}$ | $C_1(S_1, \overline{S}_1) = (t_{01} + t_{02} + t_{03} + t_{04} + t_{06}) + t_{57}$  
$\overline{S}_1 = \{1, 2, 3, 4, 6\}$ |
For \( q = 2, 3, 4 \) and 5, the \( q \)-locale networks and computational details are listed below:

<table>
<thead>
<tr>
<th>( q )</th>
<th>( S_q )</th>
<th>( \overline{S}_q )</th>
<th>( C_q(S_q, \overline{S}_q) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>( {1, 6, \cdots} )</td>
<td>( {2, 3, 4, 5} )</td>
<td>((t_{01} + t_{02} + t_{03} + t_{04} + t_{05}) + (t_{17} + t_{67}) + (w_{12} + w_{13} + w_{14} + w_{15} + w_{26} + w_{36} + w_{46} + w_{56}) = 163)</td>
</tr>
<tr>
<td>2</td>
<td>( {2, 6} )</td>
<td>( {1, 3, 4, 5} )</td>
<td>((t_{01} + t_{02} + t_{03} + t_{04} + t_{05}) + (t_{27} + t_{67}) + (w_{12} + w_{13} + w_{24} + w_{25} + w_{16} + w_{26} + w_{36} + w_{46} + w_{56}) = 139)</td>
</tr>
<tr>
<td>2</td>
<td>( {3, 6} )</td>
<td>( {1, 2, 4, 5} )</td>
<td>((t_{01} + t_{02} + t_{03} + t_{04} + t_{05}) + (t_{37} + t_{67}) + (w_{13} + w_{23} + w_{34} + w_{35} + w_{16} + w_{26} + w_{36} + w_{46} + w_{56}) = 173)</td>
</tr>
<tr>
<td>2</td>
<td>( {4, 6} )</td>
<td>( {1, 2, 3, 5} )</td>
<td>((t_{01} + t_{02} + t_{03} + t_{04} + t_{05}) + (t_{47} + t_{67}) + (w_{14} + w_{24} + w_{34} + w_{35} + w_{16} + w_{26} + w_{36} + w_{46} + w_{56}) = 159)</td>
</tr>
<tr>
<td>2</td>
<td>( {5, 6} )</td>
<td>( {1, 2, 3, 4} )</td>
<td>((w_{01} + w_{02} + w_{03} + w_{04}) + (w_{57} + w_{67}) + (v_{15} + v_{16} + v_{25} + v_{26} + v_{35} + v_{36} + v_{45} + v_{46}) = 135)</td>
</tr>
</tbody>
</table>

Table 5. The computational details of the cut approach for the \( q \)-locale network
Hence, the optimum layout vector is (6, 5, 4, 1, 3, 2), and the minimum flow cost is the sum of flow from the input station to every workstation \((\sum_{i=1}^{6} t_{0i} = 20 + 5 + 12 + 14 = 51)\), from every workstation to the output station \((\sum_{j=1}^{6} t_{ij} = 14 + 10 + 15 + 12 = 51)\), and the flows from the minimum cuts \((\sum_{q=1}^{5} C_q(S_q, \bar{S}_q) = 101 + 135 + 135 + 143 + 101 = 615)\), which is 717. This can be verified through direct computation of the total flow cost, i.e.,

\[
F(\alpha) = \sum_{i=0}^{n+1} \sum_{j=0}^{n+1} t_{ij} |\alpha(i) - \alpha(j)| = \sum_{i=1}^{6} t_{0i} |\alpha(i)| + \sum_{j=1}^{6} t_{ij} |7 - \alpha(i)| + \sum_{i=1}^{6} \sum_{j=1, j\neq i}^{6} t_{ij} |\alpha(i) - \alpha(j)| = 160 + 154 + (139 + 107 + 74 + 86 + 10) = 717.
\]
4. **Concluding Remarks**

This work considered the linear flow layout problem in a manufacturing system, where the material handling devices are restricted to move along the linear row in both directions. The layout problem can be transformed into the equivalent network optimization problem of finding the minimum cut for the corresponding locale networks. Based on these characterizations, an efficient solution procedure, which runs in the complexity of $O(n^3)$ is developed.

**References**


