An Analytical Method Combining Equivalent Circuit and Magnetic Circuit for BDFRG

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This paper presents an analytical method that integrates the equivalent circuit (EC) and magnetic circuit (MC) in order to analyze and design a brushless doubly-fed reluctance generator (BDFRG). The proposed method is capable of evaluating the BDFRG performance at the design stage because the terminal voltage can be predicted through the MC and the torque can be calculated from the EC. The MC is used to calculate the air gap flux density so that the flux linkage can be further obtained for the calculation of the machine performance using the EC and a set of particular operating conditions. The integration of the EC and MC provides a rapid analysis for a direct connection between the physical dimensions and the machine performances including torque, mechanical power, electric power and efficiency. The proposed analytical method is applied to two BDFRG designs. One is a manufactured prototype. Finite element analysis and experiments are used to verify the analytical results.

Index Terms—equivalent circuit; magnetic circuit; BDFRG

I. INTRODUCTION

Brushless Doubly-Fed machines or generators (BDFM or BDFG) have been focus of several studies in recent years for renewable energy applications because they have no brush gear maintenance and use only a partially rated power electronic inverter. The reluctance rotor type of the BDFG (the brushless doubly-fed reluctance generator - BDFRG) has the advantages of a simple structure and no rotor copper loss. It is the subject of this study. Various analytical methods, which utilize equivalent circuits (ECs) [1-3] and magnetic circuits (MCs) [4-8] have been studied separately for the BDFRG. The EC methods are capable of estimating the machine performance provided that the machine parameters, such as the leakage inductances, are estimated or determined (this is usually difficult in the early stage). In contrast, the MC approach can be used to obtain a first pass design based on a prescribed specification, but it may not be able to predict the machine performance.

The d-q axis EC model for a variable-speed doubly-excited AC reluctance machine was discussed in [1]. To calculate the inductance of a BDFRG, the space vector model of the machine under a steady state was derived [2]. This is a simplified per-phase EC with two stator windings pre-defined as the primary and secondary. In [3], a primary and secondary type of EC was developed to model a 2/6 pole BDFRM, where the induced voltages from the coupling, or mutual, inductances were individually considered. All these models did not consider magnetic characteristics such as saturation so that the estimation of machine performance may not be accurate.

A magnetic circuit (MC) was proposed for the doubly-fed doubly-salient machine in [4]. In [5], the flux produced by the stator winding currents was calculated for each stator tooth using a magnetic circuit model. This took saturation into account. A design approach for the permanent magnet (PM) machine, based on a MC, was discussed in [6], where a segmented PM model was proposed. To calculate the flux density, the relative position of the stator and rotor teeth should be considered so that the coupling behavior between the stator and rotor teeth can be modeled. Therefore, a flux coupling matrix was proposed along with a developed MC for the BDFRG. An MC was also developed for BDFM in [7]. Further study of the induction and reluctance rotor was discussed in [8]. The above research may consider saturation; the flux density is a major factor that needs to be calculated in order to evaluate the performance correctly.

This paper presents a method that allows a developed time-stepped MC to be integrated with an EC for a BDFRG design. The analysis and design process possesses the electric power output calculation to satisfy the system requirement and saturation prediction for a first pass machine design. This helps to produce a more accurate design for the BDFRG prior to finite element simulation or prototyping and thus the development cost and time can be significantly reduced.

II. METHODOLOGY

A. Integrated calculation process

The analytical process is presented in the flow chart shown in Fig. 1. This flow chart can be divided into two major parts: the MC and the EC. Each part will be detailed in the following sections. The flux density is firstly calculated by the MC based on preset winding currents. These are then passed on to the EC for calculation of torque. The mechanical, electrical power and efficiency can then be further obtained.

B. Case study of the BDFRG

An initial and ideal design model is applied to verify the proposed method, as shown in Fig. 2. The dimensions of the BDFRG are tabulated in Table I. The rotor consists of six salient poles and each pole is fabricated with 6 axial ducts. The control winding (8-pole secondary winding) and power winding (4-pole primary winding) are both 3-phase and wye connected.

Finite element analysis (FEA) simulation (using ANSYS-Maxwell software) is used to investigate the operating range
for the initial design. The rotor speed and frequency of the control and power windings are set; these are functions of the excitation and speed of the doubly-fed machine, which are tabulated in Table II. From the FEA, the operating region of the BDFRG is shown in Fig. 3. The rated operation is at 1380 rpm. Under this condition the BDFRG gives 541.5 W of electric power output. This is then applied to the proposed method for verification.

The terminal voltage is then applied to the 8p power. The flux power 1500 1700 under this condition the BDFRG gives 541.5 a lux. Tom 1300

![Fig. 1 Integrated calculation process from MC and EC.](image1)

![Fig. 2. Quarter lamination of the initial design BDFRG (white part represents the electric steel).](image2)

![Fig. 3. Operating range of the prototype BDFRG.](image3)

\[ \text{Efficiency} = \frac{P_{\text{electric}} - P_{\text{mechanical}}}{P_{\text{electric}}} \]

\[ \text{Power (W)} = \begin{cases} 4p & \text{power} \\ 8p & \text{power} \\ \text{electric power} \\ \text{mechanical power} \end{cases} \]

\[ \text{Speed (rpm)} = \begin{cases} 1200 & \text{rpm} \\ 1400 & \text{rpm} \\ 1600 & \text{rpm} \\ 1800 & \text{rpm} \end{cases} \]

\[ \text{Efficiency} = \begin{cases} 0 & \% \\ 20 & \% \\ 40 & \% \\ 60 & \% \\ 80 & \% \end{cases} \]

\[ \text{Table I MAIN MACHINE DIMENSIONS [MM]} \]

<table>
<thead>
<tr>
<th>Item</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of air gap</td>
<td>0.5</td>
<td>Height of the stator shoe</td>
</tr>
<tr>
<td>Stator slot number</td>
<td>48</td>
<td>Width of the rotor tooth</td>
</tr>
<tr>
<td>Stack length</td>
<td>81</td>
<td>Width of the rotor tooth</td>
</tr>
<tr>
<td>Coil turns (4- and 8-pole)</td>
<td>15</td>
<td>Width of the rotor rim</td>
</tr>
<tr>
<td>Stator tooth width</td>
<td>3.8</td>
<td>Height of the rotor shoe</td>
</tr>
<tr>
<td>Width of the stator yoke</td>
<td>10</td>
<td>Outer slot radius</td>
</tr>
<tr>
<td>Rotor radius</td>
<td>70</td>
<td>Height of the stator slot</td>
</tr>
</tbody>
</table>

\[ \text{Table II OPERATING CONDITION} \]

<table>
<thead>
<tr>
<th>Item</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>rotor speed [rpm]</td>
<td>1380</td>
<td></td>
</tr>
<tr>
<td>4-pole</td>
<td>Phase voltage [V]</td>
<td>100</td>
</tr>
<tr>
<td>winding</td>
<td>Phase current [A]</td>
<td>5.3</td>
</tr>
<tr>
<td>Frequency [Hz]</td>
<td>60</td>
<td></td>
</tr>
<tr>
<td>8-pole</td>
<td>Phase voltage [V]</td>
<td>179</td>
</tr>
<tr>
<td>winding</td>
<td>Phase current [A]</td>
<td>12.1</td>
</tr>
<tr>
<td>Frequency [Hz]</td>
<td>78</td>
<td></td>
</tr>
</tbody>
</table>

C. Magnetic circuit (MC) calculation

The flux density that is governed by the reluctance matrix and the external current sources is calculated through an iterative algorithm (Fig. 1); the operating point of the electric steel converges to a point on the material B/H curve. This indicates that the reluctances in the MC are variable, as shown in Fig. 4. The components in the MC include the air-gap magnetic reluctance \( R_{p} \), the reluctance of \( n^{th} \) stator and rotor tooth \( R_{s,n} \) and \( R_{r,n} \), the reluctance of \( n^{th} \) stator and rotor yoke \( R_{s,y} \) and \( R_{r,y} \); \( \mu_{0} \) is the permeability of free space and \( \mu_{iron} \) is iron relative permeability. The leakage reluctances of the shoe gap in the stator and rotor are \( R_{s} \) and \( R_{r} \), respectively. The relationships (i.e., flux) between the magnetomotive force (MMF) and permeances (or reluctances) of the rotor and stator need to be determined. To solve the flux flowing through the \( n^{th} \) stator tooth \( \phi_{n} \), a nodal analysis algorithm in [7-8] is used. Thus each stator tooth flux density is determined. Details of the magnetic circuit model can be found in [8]. The flux density is calculated at each time-step, and the flux linkage of each winding set is worked out by summing the flux in each stator tooth spanned by a winding coil. The terminal voltage can be calculated by taking the derivative of the flux linkage.

D. Equivalent circuit (EC) calculation

The flux density is first calculated from the MC. The flux density linking both the control and power windings and the fundamental components are separated from the original calculation of the flux density through spectrum analysis. These are used by the EC to calculate the flux linkage and torque. For general reluctance machines, a steady-state p-s
(primary-secondary) EC in the rotor reference frame was proposed in [9], as shown in Fig. 5 (reproduced from [2]). The fundamental air gap flux linkage phasor \( \bar{\lambda}_{ps} \) is defined as:

\[
|\bar{\lambda}_{ps}| = \frac{2}{\pi} \overline{B} r_p j N_{ps,ph} K_{n1}
\]

where \( \overline{B} \) is the magnitude of the fundamental component extracted from Fourier expansion of the air gap flux density. The flux density can be calculated from the MC as described in Section II.C. Fig. 6 illustrates that the MC calculation and the FEA agree well, and thus the flux density obtained from the FEA is used here. \( r_p \) is the pole pitch of the winding set, \( N_{ps,ph} \) is the number of turns in series per phase, and \( K_{n1} \) is the first order of the winding factor, which is defined in [10] as:

\[
K_{n1} = \frac{1}{N_{ps,ph}} \sum_{k=1}^{N_{ps,ph}} e^{-j k \pi}
\]

where \( N_{ps,ph} \) is the number of coils per phase, and \( n \) is 1 for first order. The torque \( T_{ps} \) derived from the primary and secondary windings is expressed as [11]:

\[
T_{ps} = \frac{3}{2} \left( P_s + P_j \right) \cdot \bar{\lambda}_{ps} I_p \sin \phi
\]

where \( \bar{\lambda}_{ps} \) is the fundamental flux linkage of the secondary winding, \( I_p \) is the primary phase current, and \( \phi \) is the phase difference between induced voltage and current.

![Fig. 5](image)

**Fig. 5.** (a) the equivalent circuit (reproduced from [2]); and (b) steady state phasor diagram of BDFRG.

![Fig. 6](image)

**Fig. 6.** Flux density comparison of 48 stator tooth: FEA simulation and MC calculation.

### III. Results Discussion

Two cases are individually discussed to verify the proposed method: an initial (ideal) design model and the experimental prototype.

#### A. Initial design model

To validate the proposed method, the initial BDFRG design is used for comparison between the FEA and analytical calculation. The current sources under rated conditions are first used in the MC, and the flux density and terminal voltage are determined, as shown in Figs. 6 and 7. The result shows that the flux density can be accurately calculated and the fundamental term of the terminal voltage gives 150 V and 95 V for 4-pole and 8-pole windings, respectively. The voltage calculation presents some high order harmonics compared to the simulation. This may be caused by the coupling factor of the rotor and stator in the MC [8] which is not comprehensively considered. Nevertheless, the trends of the calculation and simulation are quite similar.

![Fig. 7](image)

**Fig. 7.** Node voltage of prototype from MC: (a) 8-pole winding; and (b) 4-pole winding.

The flux density is then passed to the EC for torque calculation at certain operating conditions and the results are summarized in Table III. Compared to the FEA simulation that gives a torque of -13.3 Nm (as the steady state result shown in Fig. 8), the calculation also produces a similar result of \(-13.61\) Nm \((T_{ps})\). The calculation result is listed in Table III. Generally, the EC calculation is sufficiently accurate for performance prediction.

![Fig. 8](image)

**Fig. 8.** Torque of initial design machine from FEA simulation.
TABLE III CALCULATION RESULTS

<table>
<thead>
<tr>
<th>Item</th>
<th>8-p winding</th>
<th>Item</th>
<th>8-p winding</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_r$</td>
<td>12.3 A</td>
<td>$r_p$</td>
<td>0.06</td>
</tr>
<tr>
<td>$B$</td>
<td>0.64 T</td>
<td>$N$</td>
<td>240</td>
</tr>
<tr>
<td>$L_{sw}$</td>
<td>0.08 m</td>
<td>$K_{c1}$</td>
<td>0.97</td>
</tr>
<tr>
<td>$\delta_{m}$</td>
<td>0.45 Wb</td>
<td>$I_{ps}$</td>
<td>-13.61 Nm</td>
</tr>
</tbody>
</table>

B. Experimental prototype

The experimental setup is presented in Fig. 9 and Table IV. This is only a simple experiment and the prototype is not well designed, and the torque is quite small. Thus, this paper only focuses on the MC comparisons for the experimental case. Fig. 10 illustrates that the MC calculations give a smaller value than that in the simulation and the experimental case. The errors in the flux density calculation may be due to the reluctance variations. The coupling factor variations between the windings may also cause errors. Generally, the proposed MC is considered to be sufficiently accurate for prediction of the flux density.

TABLE IV OPERATING CONDITION OF EXPERIMENTAL PROTOTYPE

<table>
<thead>
<tr>
<th>Item</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>rotor speed</td>
<td>rpm</td>
<td>1200</td>
</tr>
<tr>
<td>4-p winding</td>
<td>Phase voltage (V)</td>
<td>40</td>
</tr>
<tr>
<td>4-p winding</td>
<td>Phase current (A)</td>
<td>0.8</td>
</tr>
<tr>
<td>8-p winding</td>
<td>Frequency (Hz)</td>
<td>60</td>
</tr>
<tr>
<td>8-p winding</td>
<td>Phase voltage (V)</td>
<td>179</td>
</tr>
<tr>
<td>8-p winding</td>
<td>Phase current (A)</td>
<td>10</td>
</tr>
<tr>
<td>8-p winding</td>
<td>Frequency (Hz)</td>
<td>60</td>
</tr>
</tbody>
</table>

Fig. 9 The experimental prototype BDFRG: (a) Quarter model of rotor and stator, (b) 48-slot stator and (c) 6 pole segmented reluctance rotor.

IV. CONCLUSIONS

The equivalent circuit and magnetic circuit methods have been introduced in this paper. Calculation examples, based on the equivalent circuits, are presented. The flux density was calculated and the magnetic circuit was discussed, which was examined with FEA simulation. With the flux density from the magnetic circuit, the torque, electrical, mechanical power and efficiency of the BDFRG can be measured or calculated from the integrated process. Further work will be to focus on improving the reluctance model of the air gap and rotor.

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