Analysis of Inductive Coupling Coils for Extending Distances of Efficient Wireless Power Transmission

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Abstract — This paper proposed an approach to extend the transmission distances of wireless power transmission (WPT) systems under the constraint of the coil size and optimized delivered power and efficiency. Based on the reflected load theory, the inductive coupling coils can be modelled as an equivalent circuit for simple analysis and be extended to multiple-coil wireless power transmission systems. According to this proposed approach, power transmission efficiency (PTE), power delivered to load (PDL), and geometric factors of the coil are taken into consideration simultaneously as the proposed figure-of-merit (FoM) to optimize the performance of the wireless power transmission systems. Finally, a practical 4-coil WPT was implemented to verify the theory, and based on the FoM, the most efficient design parameters of WPT were identified. The optimal distance was increased by 50%, and the PTE remains more than 78%.

Index Terms — Inductive Coupling, Reflected Load Theory, Wireless Power Transmission.

I. INTRODUCTION

The wireless power transmission (WPT) technologies and applications have attracted attention in recent years. Strong coupled magnetic resonance theory [1] was proposed in 2007 and demonstrated a feasible high-efficient WPT system. Magnetic resonance-based WPT is a directional coupled type of near-field power transmission within a short distances. The design of the inductive coupling coils is crucial to the wireless power transmission. To quantify the characteristics of the coils, the whole systems are primarily analyzed based on two kinds of theories: the coupled mode theory [2] and the reflected load theory [3][4]. Reflection load theory quantifies the mutual inductance between the coils through circuit theories while coupled mode theory observes the power exchange process between two resonators in physics. This paper applies the reflected load theory to analyze the proposed coil system.

To estimate the transmission performance of the entire coil system, two major aspects are power transmission efficiency (PTE) and power delivered to load (PDL) [4][5]. Both PTE and PDL depend on the coil quality factors and the coupling coefficients between the coils. Furthermore, quality factor and coupling coefficient will be affected by the coil structure, size, and distance between coils. Optimal parameters of PTE and PDL cannot be achieved at the same time and need some tradeoff. When one of major performance requirement, such as PTE or PDL, is optimized, the other may suffer the dramatic loss. In [5], the authors addressed a general figure-of-merit (FoM) for designing high-performance inductive power transmission links by considering both PTE and PDL to solve this problem. This paper further proposed a FoM measure for estimating the coil transmission performance by including the coil size, transmission distances, PTE and PDL. Such a FoM considers not only the electrical aspects but also the physical aspects including the geometric parameters and the transmission distance.

II. THEORY OF INDUCTIVE COUPLING COILS AND FoM

To analyze inductive coupling coils, the reflected load theory was applied for quantifying the mutual inductance between the coils. Fig. 1 (a) presents the equivalent circuit for the two-coil WPT system. The primary loop (coil #2) and the secondary loop (coil #3) compose of the resonators modeled by an inductance and a capacitance. $k_{23}$ is the coupling coefficient between the coil #2 and coil #3. The power can be delivered through the coupling to the secondary loop.

![Fig. 1. Two-coil equivalent lumped circuit model.](image-url)

Based on this theory, the mutual inductance between the coils can be simplified into an equivalent reflected impedance ($k_{23}^2L_2C_{ref}$ and $R_{ref}$) on the primary side coil from the secondary side winding. The inner resistance of the
secondary coil $R_3$ can be equivalently converted to a shunt $R_{3} = \bar{Q} R_3$. Fig. 1(b) illustrates the whole circuit with the equivalent reflected impedance. When the circuit operates at the resonant frequency, the reactive parts are cancelled and negligible as a pure resistance. The PTE $\eta_{2-coil}$ can be calculated in (1), implying that $\eta_{2-coil}$ is determined by $R_n$, $R_2$, $R_{ref}$ and the divided voltage between $R_{P2}$ and $R_L$:

$$\eta_{2-coil} = \frac{R_{ref}}{R_2 + R_{ref}} \frac{R_{P2}}{R_2 + R_{ref} + R_L} = \frac{k_1^2 Q_2 Q_{ref}}{1 + k_2^2 Q_2 Q_{ref} Q_L}$$

(1)

where $Q_{2L} = Q_2 Q_L/(Q_2 + Q_L)$ and $Q_2 = R_2/\omega L_2$. The power delivered to load PDL ($P_{2-coil}$) can be calculated in (2):

$$P_{2-coil} = \frac{V^2}{2(R_2 + R_{ref})} \frac{Q_2 Q_{ref}}{Q_2 Q_L} = \frac{V^2}{2(R_2 + R_{ref})} \frac{k^2_1 Q_2 Q_{ref} Q_L}{1 + k_2^2 Q_2 Q_{ref} Q_L}$$

(2)

From (2), the maximal $P_{2-coil}$ occurs at $R_{ref} = R_2$, however, in such a condition the $\eta_{2-coil}$ is less than 50%. This indicates that there are tradeoff between PDL ($P_{2-coil}$) and PTE ($\eta_{2-coil}$), and the optimal parameters should be defined and designed according to different conditions and criteria. Meanwhile, from (1) and (2), the parameters related to PTE ($\eta_{2-coil}$) and PDL ($P_{2-coil}$) include the quality factor of the coil $Q$ and the coupling coefficient between coils $k$. The coil quality factor $Q$ is affected by the radius of the coil and the wire properties. The coupling coefficient $k$ between coils is dependent on the coil sizes and the separation distance. Therefore, the coil size and the distance should be taken into account not only for the designed parameters but also a performance estimate. To evaluate and compare a two-coil WPT system, a $FOM_{2-coil}$ is proposed as (3):

$$FOM_{2-coil} = \frac{d^2_{1,2}}{\bar{R}_{P2}} \times \eta_{2-coil} \times P_{2-coil}$$

(3)

The first term of $FOM_{2-coil}$ is the square of the distance between the two coils divided by the product of the radius of the two coils. $\eta_{2-coil}$ is the PTE whose value is less than 100%. $P_{2-coil}$ is the PDL whose unit is watt. Therefore, the unit of the $FOM_{2-coil}$ is watt. The similar principle and analysis can be extended to multiple-coil systems. The equivalent circuit for a multi-coil WPT system is shown in Fig. 2.

![Diagram of Multi-coil Equivalent Lumped Circuit Model](image)

**Fig. 2. Multi-coil equivalent lumped circuit model.**

The non-adjacent mutual coupling is sufficiently weak and can be negligible. Only the adjacent mutual coupling needs to be considered. From the load stage, the impedance can be reflected reversely from the latter stages to the former stages step by step until the overall equivalent impedance is converted at the primary side. For an m-coil WPT system, let the impedance at the $i$-th coil reflected from the $(i+1)$-th coil be $R_{ref,i,i+1}$ (assume $i + 1 \leq m$).

$\eta_{i,i+1}$ that transmits from the $i$-th coil to the $(i+1)$-th coil can be obtained by the voltage division ratio of the inner resistance $R_i$ of the $i$-th coil and the reflected resistance $R_{ref,i,i+1}$ of the $(i+1)$-th coil as (4):

$$\eta_{i,i+1} = \frac{R_{ref,i,i+1}}{R_i + R_{ref,i,i+1}}$$

(4)

For a multiple-coil system, the whole efficiency $\eta_{m-coil}$ is the product of a sequence $\eta_{i,i+1}$ multiplied by the voltage division of the inner resistance of the $m$-th coil over the load impedance as (5):

$$\eta_{m-coil} = \prod_{i=1}^{m-1} \eta_{i,i+1} \frac{Q_{ref}}{Q_L}$$

(5)

The power delivered to the load ($P_{m-coil}$) of the entire m-coil WPT system can be obtained in (6):

$$P_{m-coil} = \frac{V^2}{2(R_1 + R_2 + \cdots + R_m)} \eta_{m-coil}$$

(6)

The overall figure-of-merit $FOM_{m-coil}$ for the multiple-coil WPT can be extended from (3) and be expressed in (7):

$$FOM_{m-coil} = \left[\sum_{i=1}^{m-1} d_{i,i+1}\right]^2 \times \eta_{m-coil} \times P_{m-coil}$$

(7)

The first term of $FOM_{m-coil}$ can be regarded as the square of the total transmission distance and is divided by the square of the geometric mean of the coil radii.

### III. Experimental Results and Discussion

A four-coil WPT system was implemented for verification of the system performance. SWG #16 whose wire diameter is 1.6 mm was used for the specifications of the wound solenoid coils. The characteristics of the parameters listed in TABLE I. Parameters $a$, $r$, $N$ represent the wire radius, the radius of the coil, and the turn number. The coil inductance $L_i$ and internal resistance $R_i$ are measured by using the Agilent E5071C network analyzer. The measured Q values are lower than the theoretical $Q$ values due to the inter-connection loss.

The distances between the source and load coils are fixed: $d_{1,2} = d_{3,4} = 0.5$ cm where $d_{1,2}$ represents the distance between coil #1 and coil #2, and so is other distance representation. The distance $d_{3,1}$ is varied from 0 cm to 10 cm to measure the PTE and PDL of the four-coil WPT system. As for the PDL measurement, the network analyzer transmits 5-dBm power from the coil #1 as the source. Coil #4 (load loop) is connected to the Agilent U2000A power meter to measure the PDL. The entire experimental system architecture is shown in Fig. 3.
Fig. 3. Measurement setup.

**TABLE I**

<table>
<thead>
<tr>
<th>Parameters of Four-Coil Links</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_0 (MHz) )</td>
<td>13.56</td>
</tr>
<tr>
<td>( R_1 (\Omega) )</td>
<td>50</td>
</tr>
<tr>
<td>( R_2 (\Omega) )</td>
<td>50</td>
</tr>
<tr>
<td>( L_1 (\mu H) )</td>
<td>0.111 / 0.136</td>
</tr>
<tr>
<td>( Q_1 )</td>
<td>318.24 / 141.21</td>
</tr>
<tr>
<td>( a_1 (mm), r_1 (mm), N_1 )</td>
<td>0.8, 25, 1</td>
</tr>
<tr>
<td>( L_2 (\mu H) )</td>
<td>0.996 / 0.848</td>
</tr>
<tr>
<td>( Q_2 )</td>
<td>954.71 / 344.14</td>
</tr>
<tr>
<td>( a_2 (mm), r_2 (mm), N_2 )</td>
<td>0.8, 25, 3</td>
</tr>
<tr>
<td>( L_3 (\mu H) )</td>
<td>0.996 / 0.840</td>
</tr>
<tr>
<td>( Q_3 )</td>
<td>954.71 / 332.74</td>
</tr>
<tr>
<td>( a_3 (mm), r_3 (mm), N_3 )</td>
<td>0.8, 25, 3</td>
</tr>
<tr>
<td>( L_4 (\mu H) )</td>
<td>0.111 / 0.133</td>
</tr>
<tr>
<td>( Q_4 )</td>
<td>318.24 / 166.91</td>
</tr>
<tr>
<td>( a_4 (mm), r_4 (mm), N_4 )</td>
<td>0.8, 25, 1</td>
</tr>
</tbody>
</table>

Note: Two values include theoretical calculation / measurement.

Fig. 4 shows the measured \( \eta_{4\text{-coil}} \) compared with measured FoM\(_{4\text{-coil}}\) of the 4-coil WPT system. When the distance \( d_{2,3} \) is less than 3 cm, there is approximately 6% difference between measured and theoretical prediction values of \( \eta_{4\text{-coil}} \). This may come from the neglect of the non-adjacent mutual coupling whose contribution increases in practice at the short distances, implying this effect cannot be ignored. Moreover, some errors may be caused by the distance inaccuracy and misalignment of the co-axis for the multiple coils. Generally speaking, the theoretical \( \eta_{4\text{-coil}} \) remains more than 90% when the distance \( d_{2,3} \) is smaller than 3 cm, and degrades significantly after the distance \( d_{2,3} \) is larger than 4 cm. In other words, to achieve the optimal operation distance according to the high efficiency criteria, the distance \( d_{2,3} \) can be chosen up to 3 cm. At this distance, the measured \( \eta_{4\text{-coil}} \) is 86.3% and has 3% error from the theoretical \( \eta_{4\text{-coil}} \). However, according to the proposed FoM\(_{4\text{-coil}}\) design, the FoM\(_{4\text{-coil}}\) achieves its optimal performance at the distance of 4.5 cm. This indicates that our system can be designed to operate in an extended distance (increased by 50%), but still maintain 78% of \( \eta_{4\text{-coil}} \) for maximal FoM\(_{4\text{-coil}}\).

**IV. Conclusion**

In this study, the reflection load theory is applied to analyze inductive coupling coils for wireless power transmission system. Considering transmission efficiency, power, and coil geometry, this paper proposes a FoM to measure the performance of the WPT system. The theoretical results are verified by an implemented 4-coil WPT system. Designed according to the proposed FoM, the wireless power transmission system can be operated in better condition including a further distance with more than 78% of PTE. In the future, the system design can take other factors into further consideration including operation bandwidth, power consumption, and safety regulations. Each of these crucial factors can be quantified by FoM to estimate and optimize the comprehensive WPT features.

**V. Acknowledgement**

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**Reference**


