Decoupling six effective parameters of anisotropic materials using Stokes polarimetry

PhD student: PHAM, THI-THU-HIEN
Advisor: Prof. LO, YU-LUNG

National Cheng Kung University
Department of Mechanical Engineering
Introduction

Tissues (turbid media) $\Rightarrow$ Optics $\Rightarrow$ Light scattering $\Rightarrow$ Potential applications (medical diagnosis)

Random scattering on a rough surface

Scattering on impurities in the volume
Depolarization

Polarized light → Unpolarized light

Polarization & Depolarization

Associate

Scattering, Retardance, Diattenuation

Introduction
Some current techniques

Monte Carlo

Simulate

measure

The system

Polyacrylamide phantoms

Polystyrene microspheres

Sucrose

Optical properties

• Linear retardance ($\beta$)
• Optical rotation angle ($\gamma$)
• Diattenuation (D)
• Depolarization coefficient ($\Delta$)

The system

Monte Carlo

Simulate

measure

The system

Polyacrylamide phantoms

Polystyrene microspheres

Sucrose

Optical properties

• Linear retardance ($\beta$)
• Optical rotation angle ($\gamma$)
• Diattenuation (D)
• Depolarization coefficient ($\Delta$)

Their study did not show enough nine parameters of characteristics of a bio-sample.

One famous group is from Canada (Ghosh et al or Wood et al)
Some current techniques

Other famous group is from Portland and USA (Prahl et al): many papers

The system

Integrating sphere

measure

enhance

The system

Bio-samples

Tissue chromophores (water, dry tissue and blood)

Polystyrene microspheres

Bovine muscle

Optical properties

• The absorption coefficient ($\mu_a$)
• The scattering coefficient ($\mu_s$)
• The anisotropy factor ($g$)

Their study did not show enough nine parameters of characteristics of a bio-sample.
Some current techniques

Other famous group is from USA (Cameron et al): many papers had published

Mueller matrix & Stokes polarimeter

The system → measure

Bio-samples

Polystyrene microspheres (different diameter)

Melanoma

Rat skin

Optical properties

• The scattering coefficient ($\mu_s$)
• The image of Mueller matrix

The system diagram is shown with various components like Argon Laser, Mirrors, Polarizers, Variable Retarders, etc.

The image of Mueller matrix for a complex polystyrene mixture.

Their study did not show enough nine parameters of characteristics of a bio-sample.
The Stokes vector of the output light:

\[
S = \begin{bmatrix}
S_0 \\
S_1 \\
S_2 \\
S_3
\end{bmatrix} = \begin{bmatrix}
I_x + I_y \\
I_x - I_y \\
I_{45^\circ} - I_{-45^\circ} \\
I_{rcp} - I_{lcp}
\end{bmatrix}
\]

- \( S_0 \): the total light intensity;
- \( S_1 \): the intensity difference between horizontally and vertically polarized components
- \( S_2 \): the intensity difference between +45\(^\circ\) and -45\(^\circ\) polarized components
- \( S_3 \): the intensity difference between right- and left-circularly polarized components

The Stokes vector of the output light:

\[
S = \begin{bmatrix}
S_0 \\
S_1 \\
S_2 \\
S_3
\end{bmatrix} = \begin{bmatrix}
m_{11} & m_{12} & m_{13} & m_{14} \\
m_{21} & m_{22} & m_{23} & m_{24} \\
m_{31} & m_{32} & m_{33} & m_{34} \\
m_{41} & m_{42} & m_{43} & m_{44}
\end{bmatrix}\begin{bmatrix}
\hat{S}_0 \\
\hat{S}_1 \\
\hat{S}_2 \\
\hat{S}_3
\end{bmatrix} = M\hat{S}
\]

Stokes vectors

Mueller matrices

The polarization state of the light (optical sample)
Measuring *six parameters* of characteristics in materials:

- Principal axis angle ($\alpha$)
- Retardance ($\beta$)
- Optical rotation angle ($\gamma$)
- Diattenuation axis angle ($\theta_d$)
- Diattenuation ($D$)
- Circular diattenuation ($R$)

\[\{\text{Linear birefringence (LB)} \quad \text{Circular birefringence (CB)} \quad \text{Linear diattenuation (LD)} \quad \text{Circular diattenuation (CD)}\]
The six effective optical parameters of an anisotropic material:

- Principal axis angle & retardance of LB,
- Optical rotation of CB,
- Diattenuation axis angle & diattenuation of LD,
- Circular diattenuation of CD

\[ \mathbf{M} = [\mathbf{M}_R][\mathbf{M}_D] \]

\[ \mathbf{M}_R = [\mathbf{M}_{lb}][\mathbf{M}_{cb}] \quad \mathbf{M}_D = [\mathbf{M}_{ld}][\mathbf{M}_{cd}] \]

\[ \mathbf{S}_c = \begin{bmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{bmatrix}_c = [\mathbf{M}_{lb}][\mathbf{M}_{cb}][\mathbf{M}_{ld}][\mathbf{M}_{cd}] \hat{\mathbf{S}}_c = \begin{pmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ m_{41} & m_{42} & m_{43} & m_{44} \end{pmatrix} \begin{pmatrix} \hat{S}_0 \\ \hat{S}_1 \\ \hat{S}_2 \\ \hat{S}_3 \end{pmatrix}_c \]
Mueller matrices of six parameters

\[
M_{lb} = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & \cos(4\alpha)\sin^2(\beta/2) + \cos^2(\beta/2) & \sin(4\alpha)\sin^2(\beta/2) & \sin(2\alpha)\sin(\beta) \\
0 & \sin(4\alpha)\sin^2(\beta/2) & -\cos(4\alpha)\sin^2(\beta/2) + \cos^2(\beta/2) & -\cos(2\alpha)\sin(\beta) \\
0 & -\sin(2\alpha)\sin(\beta) & \cos(2\alpha)\sin(\beta) & \cos(\beta)
\end{pmatrix}
\]

\[
M_{cb} = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & \cos(2\gamma) & \sin(2\gamma) & 0 \\
0 & -\sin(2\gamma) & \cos(2\gamma) & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
\]

\[
M_{cd} = \begin{pmatrix}
1 + R^2 & 0 & 0 & 2R \\
0 & 1 - R^2 & 0 & 0 \\
0 & 0 & 1 - R^2 & 0 \\
2R & 0 & 0 & 1 + R^2
\end{pmatrix}
\]

\[
M_{ld} = \begin{pmatrix}
\frac{(1+\frac{1-D}{1+D})}{2} & \cos(2\theta_d)(1-\frac{1-D}{1+D}) & \cos(2\theta_d)(1-\frac{1-D}{1+D}) & 0 \\
\cos(2\theta_d)(1-\frac{1-D}{1+D}) & \frac{(1+\sqrt{\frac{1-D}{1+D}})^2}{4} & \cos(4\theta_d)(1-\frac{1-D}{1+D}) & 0 \\
\sin(2\theta_d)(1-\frac{1-D}{1+D}) & \sin(4\theta_d)(1-\frac{1-D}{1+D}) & \frac{(1+\sqrt{\frac{1-D}{1+D}})^2}{4} & 0 \\
0 & \sin(4\theta_d)(1-\frac{1-D}{1+D}) & \frac{(1+\sqrt{\frac{1-D}{1+D}})^2 - \cos(4\theta_d)(1-\frac{1-D}{1+D})^2}{4} & 0 \\
0 & 0 & 0 & \sqrt{\frac{1-D}{1+D}}
\end{pmatrix}
\]
Measurement of **LB/CB** and **LD/CD**

The output Stokes vectors can be obtained as:

\[
\begin{align*}
S_{0^0} &= \begin{bmatrix} m_{11} + m_{12}, & m_{21} + m_{22}, & m_{31} + m_{32}, & m_{41} + m_{42} \end{bmatrix}^T \\
S_{45^0} &= \begin{bmatrix} m_{11} + m_{13}, & m_{21} + m_{23}, & m_{31} + m_{33}, & m_{41} + m_{43} \end{bmatrix}^T \\
S_{90^0} &= \begin{bmatrix} m_{11} - m_{12}, & m_{21} - m_{22}, & m_{31} - m_{32}, & m_{41} - m_{42} \end{bmatrix}^T \\
S_{135^0} &= \begin{bmatrix} m_{11} - m_{13}, & m_{21} - m_{23}, & m_{31} - m_{33}, & m_{41} - m_{43} \end{bmatrix}^T \\
S_{RHC} &= \begin{bmatrix} m_{11} + m_{14}, & m_{21} + m_{24}, & m_{31} + m_{34}, & m_{41} + m_{44} \end{bmatrix}^T \\
S_{LHC} &= \begin{bmatrix} m_{11} - m_{14}, & m_{21} - m_{24}, & m_{31} - m_{34}, & m_{41} - m_{44} \end{bmatrix}^T
\end{align*}
\]

\[
S = \begin{bmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ m_{41} & m_{42} & m_{43} & m_{44} \end{bmatrix} \begin{bmatrix} \hat{S}_0 \\ \hat{S}_1 \\ \hat{S}_2 \\ \hat{S}_3 \end{bmatrix} = MS
\]

**Find**

\((\alpha, \beta, \gamma, \theta_d, D, \text{and } R)\)

**Known**
Diattenuation axis angle:

\[ 2\theta_d = \tan^{-1}\left( \frac{S_{45^0}(S_0) - S_{135^0}(S_0)}{S_{0^0}(S_0) - S_{90^0}(S_0)} \right) \]

Diattenuation:

\[ D = \frac{[S_{0^0}(S_0) - S_{90^0}(S_0)]}{\cos(2\theta_d)[\sqrt{(S_{0^0}(S_0) + S_{90^0}(S_0))^2 - (S_{RHC}(S_0) + S_{LHC}(S_0))^2}]} \]

Circular diattenuation:

\[ R = \frac{[S_{0^0}(S_0) - S_{90^0}(S_0)] - [\sqrt{(S_{0^0}(S_0) + S_{90^0}(S_0))^2 - (S_{RHC}(S_0) + S_{LHC}(S_0))^2}]}{[S_{RHC}(S_0) + S_{LHC}(S_0)]} \]

Linear birefringence axis angle:

\[ \alpha = \frac{1}{2} \tan^{-1}\left( -\frac{A_{24}}{A_{34}} \right) \]

Retardance:

\[ \beta = \tan^{-1}\left( \frac{A_{34}}{\cos(2\alpha)A_{44}} \right) \]

Circular birefringence:

\[ \gamma = \frac{1}{2} \tan^{-1}\left( \frac{C_3A_{23} - C_2A_{33}}{-C_2A_{23} + C_1A_{33}} \right) \]

The analytical model enables the \textbf{full-range} measurement of the principal axis angle, optical rotation angle, diattenuation axis angle, diattenuation and circular diattenuation. However, the measurable range of the phase retardance is limited to \(0 \sim 180^\circ\).
Experimental setup for measurement of six parameters

The six input polarization states:

- Four linear polarization lights
  \[ \hat{S}_0 = [1, 1, 0, 0] \]
  \[ \hat{S}_{90} = [1, -1, 0, 0] \]
  \[ \hat{S}_{45} = [1, 0, 1, 0] \]
  \[ \hat{S}_{135} = [1, 0, -1, 0] \]

- Two circular polarization lights
  \[ \hat{S}_{RHC} = [1, 0, 0, 1] \]
  \[ \hat{S}_{LHC} = [1, 0, 0, -1] \]

The optical setup includes a He-Ne Laser, Q45°, -45°, Neutral Density Filter, Power meter detector, Sample, and Stokes Polarimeter.
Measurement of LB, CB, LD, & CD samples

The experimental results show that a good agreement is obtained between the experimental and actual values of the LB, CB, LD and CD sample.

Experimental results for (a) LB of quarter-wave plate, (b) LD of polarizer (c) CB of half-wave plate, and (d) CD of polarization controller.
The experimental results show that a good agreement is obtained between the experimental and actual values of a baked polarizer at 150°C for 100 minutes (LB and LD properties).
The experimental results show that a good agreement is obtained between the experimental and actual values of the composite sample comprising quarter-wave plate, half-wave plate and polarizer (LB, CB, and LD properties).
Measuring effective parameters in a single mode optical fiber

<table>
<thead>
<tr>
<th>Measured effective parameters</th>
<th>Shape No.</th>
<th>#1</th>
<th>#2</th>
<th>#3</th>
<th>#4</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Principal axis angle ( \alpha ) (deg.)</strong></td>
<td></td>
<td>126.27(^{0})</td>
<td>112.82(^{0})</td>
<td>114.81(^{0})</td>
<td>122.24(^{0})</td>
</tr>
<tr>
<td><strong>Retardance ( \beta ) (deg.)</strong></td>
<td></td>
<td>19.69(^{0})</td>
<td>24.02(^{0})</td>
<td>25.67(^{0})</td>
<td>132.07(^{0})</td>
</tr>
<tr>
<td><strong>Diattenuation axis angle ( \theta_d ) (deg.)</strong></td>
<td></td>
<td>8.25(^{0})</td>
<td>55.41(^{0})</td>
<td>109.5(^{0})</td>
<td>46.03(^{0})</td>
</tr>
<tr>
<td><strong>Diattenuation ( D )</strong></td>
<td></td>
<td>0.06</td>
<td>0.07</td>
<td>0.07</td>
<td>0.1</td>
</tr>
<tr>
<td><strong>Optical rotation ( \gamma ) (deg.)</strong></td>
<td></td>
<td>-22.71(^{0})</td>
<td>-22.28(^{0})</td>
<td>-23.55(^{0})</td>
<td>-62.04(^{0})</td>
</tr>
</tbody>
</table>
Measuring effective parameters in a single mode optical fiber

The schematic diagram used to measure the parameters of a LB/LD sample using a fiber-type polarimeter.

Experimental results for (a) birefringence of quarter-wave plate and (b) diattenuation of polarizer obtained using the common-path interferometer with a polarization-insensitive fiber probe.
Current work on measurement of **nine** parameters

- $e_1$: the degrees of linear depolarization
- $e_2$: the degrees of linear depolarization
- $e_3$: the degree of circular depolarization
High potential for practical applications, especially in noninvasive medical diagnosis.

The recent studies did not show enough six parameters of characteristics of a bio-sample.

Most of them did not decouple linear depolarization and circular depolarization in the scattering events.

The new proposed technique is proposed with all decoupling characteristics in six parameters which other studies do not mention before.
Applications & Development of the study

*Universal Measuring System: All parameters are decoupling (pretreatment in samples is not needed)*

Some important applications:

1. LB measurements for LCD’s compensator films……..
2. LB measurements for photoelasticity, tumors……..
3. CB measurements for diabetics……………
4. CD measurements for protein structures…………
5. LD measurements for tumors……………..
6. L-Dep, and C-Dep measurements for tumors…….
7. L-Dep, and C-Dep measurements for surface measurements…………..


Thank You!