Abstract: For robot exploration that is subject to pose uncertainty, combining multi-robot cooperative exploration strategy and active simultaneous localization and mapping (SLAM) algorithm can efficiently explore the environment while building the map from the observed data. In the paper, the exploration is stated as a constrained optimization problem and a two-phase approach is proposed. In the first phase, robots with low pose uncertainties coordinate with one another to minimize the exploration time while taking into account the uncertainties of the robot poses. Whenever the pose uncertainty of a robot exceeds a pre-defined threshold, the robot switches to the second phase by revisiting previously seen landmarks or meeting other robots. During the exploration/SLAM process, robots switch between the two phases to minimize the exploration time while maintaining the accuracy of robot poses and map. An adaptive strategy is employed to automatically adjust the threshold of the robot pose uncertainty constraints in order to prevent the robots from oscillating between the two phases. To deal with the limited communication problem, rendezvous technique is utilized by allowing robots to temporarily move out of the communication range and rejoin the group later. Simulation results are provided to verify the proposed approach.

Keywords: active SLAM, exploration, multi-robot, relocalization, adaptive, rendezvous.

1. INTRODUCTION

Exploration is the task of guiding robots in such a way that they cover the environment with their sensors in an efficient and effective manner. It is known that multi-robot systems have several advantages over single-robot systems, such as faster task completion, more efficient localization, and higher fault tolerance. In existing multi-robot exploration methods, the poses of the robots are usually assumed to be known, which is not true in many real situations. To account for unknown robot poses, several SLAM methods [2, 3, 16, 19] have been investigated to estimate the pose and build the map at the same time. However, these SLAM algorithms are passive in the sense that they only process the perceived sensor data and do not influence the motion of the mobile robot. It is noted that the path control strategy can have a substantial impact on the quality of the resulting map. Therefore, active SLAM methods have been recently proposed to efficiently explore the environment while gathering data to obtain an accurate map. However, most active SLAM methods only consider the single robot case. It is desired to extend active SLAM to the multi-robot scenarios.

In existing multi-robot exploration methods, the coordination level varies from no coordination (21)] to intensive cooperation ([16], [19], [20], [22]). To this end, several methods including integer programming [19, 20] and auction scheme [16, 22] have been proposed for the assignment of robots to targets. The idea of spreading multiple robots for better coverage has been explored in [2, 3, 16]. In [20], under the assumption of the availability of the map, a global optimization strategy for the minimization of the variance of regional waiting time and the variance of regional exploration percentage was investigated.

In multi-robot exploration, existing methods typically assume that the robots under consideration are within the communication range [13, 14, 15, 18]. More recently, a role-based exploration strategy has been proposed in [6] to allow robots to explore beyond the communication range. Robots either act as explorers or as relays to explore the environment and return information to a central command center. Rendezvous points between explorers and their corresponding relays are dynamically set during the exploration process. Extension of such a method can also be found in [7, 8] in which a new rendezvous point selection procedure and dynamic team hierarchies are adopted, respectively. Again, existing cooperative methods that allow robots to go beyond the communication range typically assume that perfect sensor data and localization are available. Existing exploration approaches can be briefly summarized in Table 1.

<table>
<thead>
<tr>
<th>Robot system</th>
<th>Cooperative</th>
<th>Slam</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single</td>
<td>No</td>
<td>Active</td>
<td>[9], [10], [11], [17]</td>
</tr>
<tr>
<td>Multiple</td>
<td>No</td>
<td>No</td>
<td>[21]</td>
</tr>
<tr>
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<td>[6], [7], [8], [13], [14], [15], [18], [20], [22]</td>
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<tr>
<td>Multiple</td>
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<td>Passive</td>
<td>[2], [3], [16], [19]</td>
</tr>
<tr>
<td>Multiple</td>
<td>Yes</td>
<td>Active</td>
<td>[12], this paper</td>
</tr>
</tbody>
</table>

The paper attempts to synergistically integrate cooperative methods and active SLAM for multiple robots to explore an unknown environment. Indeed, an active SLAM algorithm for multi-robot exploration in unknown environments was recently proposed in [12] as a scheme that combines a multi-robot exploration strategy and active SLAM algorithm to efficiently explore the environment while building the map from
the observed data. In this method, exploration is stated as a constrained optimization problem and is solved using a two-phase process. In the first (exploration) phase, robots with low pose uncertainties coordinate with one another to minimize the exploration time while taking into account the uncertainties of the robot poses. Whenever the pose uncertainty of a robot exceeds a pre-defined threshold, the robot switches to the second (relocalization) phase by returning to previously seen landmarks or meeting other robots. During the exploration/SLAM process, robots switch between the two phases to minimize exploration time while maintaining the accuracy of robot poses and map. This method is further improved in the present study. Firstly, an adaptive strategy is employed to automatically adjust the threshold of the robot pose uncertainty constraints in order to prevent the robots from oscillating between the exploration and relocalization phases. Secondly, rendezvous technique is utilized to deal with the limited communication range by allowing robots to temporary move out of the communication range and rejoin the group later. Simulations are conducted to assess the performance of the proposed approach.

The rest of this paper is organized as follows. Section 2 briefly reviews the active SLAM problem with multiple robots. Section 3 presents the details of the proposed approach. Section 4 contains the simulation results and discussions. Section 5 gives the conclusions.

2. ACTIVE SLAM PROBLEM WITH MULTIPLE ROBOTS

In SLAM problems, an autonomous vehicle with a known model starts at an unknown location in an unknown environment and then incrementally builds a map of the environment while simultaneously using that map to compute absolute vehicle location. In the multi-robot SLAM problem, the state vector comprises the poses of all robots and locations of stationary landmarks. For simplicity, a two-dimensional SLAM problem is considered. Assume that there are \( R \) robots and \( L \) landmarks, the state vector at time \( k \) is represented by:

\[
\mathbf{x}(k) = \begin{bmatrix} \mathbf{x}_1(k) & \mathbf{x}_2(k) & \ldots & \mathbf{x}_R(k) & \mathbf{q}_1 & \mathbf{q}_2 & \ldots & \mathbf{q}_L \end{bmatrix}^T
\]

(1)

where \( \mathbf{x}_r(k) \) for \( r = 1,2,\ldots,R \) is the pose of the \( r \)-th robot at time \( k \) and \( \mathbf{q}_l \) for \( l = 1,2,\ldots,L \) is the position of the \( l \)-th landmark. The pose of the \( r \)-th robot is expressed as \( \mathbf{x}_r(k) = [\mathbf{p}_r(k) \ \Theta_r(k)]^T \) where \( \mathbf{p}_r \) is the position and \( \Theta_r \) is the orientation angle. The position vectors \( \mathbf{p}_r \) and \( \mathbf{q}_l \) are sometimes expressed as \( \mathbf{p}_r = [x \ y]^T \) and \( \mathbf{q}_l = [x \ y]^T \), respectively. To describe the vehicle motion, the kinematic model for the trajectory of the front wheel of a bicycle subject to rolling motion constraints (i.e., assuming zero wheel slip) is adopted [1]. In the absence of process noise, the pose of the \( r \)-th robot is propagated according to:

\[
\mathbf{x}_r(k) = f_r(\mathbf{x}_r(k-1), \mathbf{u}_r(k))
\]

\[
= \mathbf{x}_r(k-1) + \Delta t \begin{bmatrix} v_r \sin(\gamma_r(k)) \\ v_r \cos(\gamma_r(k)) \end{bmatrix}
\]

(2)

where \( \mathbf{u}_r(k) = [v_r \ y_r]^T \) is the nominal control input for the \( r \)-th robot at time \( k \) with \( v_r \) being the velocity input and \( y_r \) being the steering angle input. In Eq. (2), \( \Delta t \) is the time step and \( B \) is wheelbase of the robot. All landmarks are assumed to be stationary. In practice, the robot model is subject to process noise and the following model is considered.

\[
\mathbf{x}(k) = f(x_{k-1}(1), u_r(k)) + w_r(k)
\]

(3)

where \( w_r(k) \) is the process noise disturbence vector to the \( r \)-th robot. The process noise vector is assumed to be an uncorrelated, zero-mean random process with a positive semi-definite covariance matrix. The process noise vectors between different robots are assumed to be uncorrelated.

Each vehicle is equipped with a sensor that is capable of measuring the relative range and angle between any individual landmark and the vehicle itself. For a range-bearing measurement at time \( k \) from the \( r \)-th robot at \( \mathbf{p}_r(k) = [x_r(k) \ y_r(k)]^T \) to the \( l \)-th landmark at \( \mathbf{q}_l = [x_l \ y_l]^T \), the observation model is given by:

\[
\mathbf{y}_{rl}(k) = \begin{bmatrix} \sqrt{(x_r(k) - x_l)^2 + (y_r(k) - y_l)^2} - \theta_r(k) + \arctan\left(\frac{y_r(k) - y_l}{x_r(k) - x_l}\right) + v_{r,l}(k) \end{bmatrix}
\]

(4)

where \( v_{r,l}(k) \) is the measurement noise vector which is also assumed to be an uncorrelated, zero-mean random process with a positive definite covariance matrix. Different measurement noise vectors are also assumed to be uncorrelated.

The goal of the exploration is to cover the whole environment in a minimum amount of time. More precisely, the robots are required to coordinate with one another to efficiently and completely explore the unknown environment in a way that minimizes the exploration time while guaranteeing the accuracy of the map. To this end, the following constrained optimization problem is formulated [12]

\[
\min \text{ exploration time}
\]

Subject to \( \text{trace} \left( P_r \right) < \alpha, r = 1,2,\ldots,R \)

(5)

where the exploration time is the time needed to
completely explore the environment, \( P_r \) is the pose error covariance of the \( r \)-th robot, and \( \alpha \) is a predefined threshold.

### 3. PROPOSED APPROACH FOR ACTIVE SLAM WITH MULTIPLE ROBOTS

#### 3.1 General framework

The proposed approach consists of two steps: one is related to the path generation by determining the target point for each robot to move towards and the other is the SLAM operation in which the pose of the robot and position of the landmarks are estimated. In the paper, the extended Kalman filter (EKF)-based SLAM algorithm is adopted. The details for implementing the multi-robot EKF-based SLAM algorithm including prediction, correction, and augmentation steps can be found in [5]. It should be noted that other filtering algorithms (particle filters, for example) can also be used.

For the determination of the optimal solution to the constrained optimization problem in Eq. (5), a two-phase process that involves exploration phase and relocalization phase was proposed in [12]. In the exploration phase, robots with low pose uncertainties cooperate with one another to explore the environment by computing the best exploration strategy as a balance of information gain, localization quality, and navigation cost. Whenever the pose uncertainty of a robot exceeds the threshold \( \alpha \), the robot switches to the relocalization phase to revisit previously seen landmarks or to meet other robots in order to relocalize itself. The robot then returns to the exploration phase to continue exploring the environment. A factor \( \lambda \) is used in the decision process to prevent the robot from oscillating between the two phases. Fig. 1 shows a diagram of this mechanism.

**Fig. 1** The two-phase process.

**Robot uncertainty > \( \alpha \)**

**Robot uncertainty < \( \lambda \times \alpha \)**

#### 3.2 Exploration phase

In the exploration phase, robots try to build a complete map in the shortest time while taking the accuracy of the map into account. The key problem in the multi-robot exploration is the selection of appropriate target points or waypoints for each robot so as to simultaneously explore different regions of the environment.

The frontier method is used to identify areas of interest which tend to lie on the edges between the explored and the unexplored regions. An occupancy grid map [4] is utilized for this purpose. Coordination among robots corresponds to the problem of finding an assignment from robots to frontiers that balances utility and cost. In order to select the target point from a list of potential candidates, the expected payoffs and costs associated with moving to the proposed locations must be calculated. Assume that there are \( T \) targets under consideration, the exploration phase is to determine the \( R \times T \) assignment matrix \( A \) which is a matrix that contains only 0 and 1. When \( A(r,t) = 1 \), the \( r \)-th robot is assigned to move to the \( t \)-th target. Each robot is assigned to exactly one target. Also, a target is not assigned to multiple robots. The assignment matrix is determined to maximize the following objective function

\[
J_{\text{exploration}} = \sum_{r=1}^{R} \sum_{t=1}^{T} A(r,t) \left[ w_I U_I(r,t) + w_w U_w(r,t) - w_C C_C(r,t) \right]
\]

In the above, \( U_I(r,t) \) is the utility of information, \( U_w(r,t) \) is the utility of localizability, \( C_C(r,t) \) is the cost of navigation, and \( w_I \), \( w_w \), and \( w_C \), respectively, are the associated weighting coefficients. The utility of information \( U_I(r,t) \) is the expected area that the \( r \)-th robot will explore at the frontier when it reaches the \( t \)-th target. The area is assessed by using an estimate of the size of the unknown area visible from the \( t \)-th target. The cost of navigation \( C_C(r,t) \) is the minimal cost path for the \( r \)-th robot to move to the \( t \)-th target. The minimal cost path can be computed efficiently using A* search. In the present study, the cost is simply the Euclidian distance between robot position and the target point. The utility of localizability \( U_w(r,t) \) is used to distinguish between target points with different localization qualities. The quality of localization at any given point is related to the uncertainty in vehicle position achievable at that point. More specifically, it is defined as the minimum covariance achievable by relocalizing a lost vehicle at a given location by observing only the landmarks visible from that location.

With respect to each robot-target pair, the objective function can be evaluated. The determination of the assignment matrix \( A \) becomes an integer programming problem. Standard tools such as the branch-and-bound algorithms can then be used to find the assignment matrix. The robots are then directed to the assigned target points.

#### 3.3 Relocalization phase

In the exploration phase, the robots choose control actions to balance the accuracy of the map and the desire to see as much of the unseen environment as possible. However, the robots can still get lost, especially when the measurement errors are large or the number of landmarks is few. To maintain the accuracy of the map, whenever the pose uncertainty of a robot exceeds a pre-defined threshold, the robot switches to the relocalization phase, that is, it revisits previously seen landmarks or meets other robots at a target point to relocalize itself. The determination of the target point is
delineated in the following. A landmark with low position uncertainty can serve as a target point. Also, instead of revisiting a previously seen landmark, the robot can also reduce its pose uncertainty by rendezvousing with other robots with good pose knowledge. In addition, if a robot (r-th robot) chooses to meet another robot (s-th robot), two situations are considered: (i) the r-th robot moves towards s-th robot while the latter stands still at its current location and (ii) both robots move towards each other.

Similar to the exploration phase, the target point for relocalization is determined by optimizing an objective function that is related to the cost of navigation $C_{N2}$, loss of efficiency $L$, utility of localizability $U_{12}$, and distance to the nearest exploration point $D$. The objective function of the r-th robot is given by

$$J_{relocalization} = w_{L12} - w_{N2}C_{N2} - w_{loss}L - w_{D2}D$$  \hspace{1cm} (7)$$

where $w_{L12}$, $w_{N2}$, $w_{loss}$, and $w_{D2}$ are weighting coefficients. The utility of localizability $U_{12}$ is used as in the exploration phase to distinguish between target points with different localization qualities. If the r-th robot and the s-th robot move towards each other, the pose uncertainty of the s-th robot at the meeting point is estimated by simulation before the localizability at the meeting point is computed. The cost of navigation $C_{N2}$ is given by the minimal cost path from the position of the r-th robot, to the selected landmark or to the position of another robot. In the present study, the cost is simply the Euclidian distance between the robot and the target point.

If the s-th robot is involved in the relocalization of the r-th robot, the original exploration task of the s-th robot is interrupted, resulting in a loss of efficiency. The loss of efficiency $L$ is assumed to be proportional to the extra time for the s-th robot for the rendezvous,

$$L = \begin{cases} 0 & \text{if the r-th robot moves towards landmark} \\
\frac{\kappa \| \mathbf{p}_r - \mathbf{p}_s \|}{v} & \text{if the r-th robot moves towards} \\
\text{the s-th robot} \\
\frac{\kappa \| \mathbf{p}_r - \mathbf{p}_s \|}{2v} & \text{if the r-th robot and the s-th robot} \\
\text{moves towards each other} 
\end{cases}$$

where $\kappa$ is a scale factor and $v$ is the velocity of the robots. When the s-th robot chooses to move towards the meeting point, the relocalization time for the r-th robot is reduced. The distance $D$ from the meeting point or landmark to the nearest exploration point must also be taken into account. If a robot chooses to come back to a previously seen area to see a good landmark, which is often located near the robot’s initial points and far away from current frontiers, it may take the robot additional time to return to conduct the exploration mission. The distance $D$ is used to assess the effort needed to go back to perform exploration.

Once the robot is forced to enter the relocalization phase, the objective function $J_{relocalization}$ in Eq. (7) as a function of all candidate target points is optimized. Once the target point is determined, the robot is controlled to move to the target point. When the control actions involve another robot, a request is sent to the designated robot to perform the relocalization operation.

### 3.4 Adaptive uncertainty threshold

In the original approach proposed in [12], the uncertainty threshold $\alpha$ in Eq. (5) is fixed. With a fixed threshold, robots may get stuck in regions with less or no landmarks by repeatedly switching between the exploration and relocalization phase as illustrated in Fig. 2, where Robot 1 gets stuck in Region 1. To overcome this problem, the uncertainty threshold is temporarily increased to allow the robot to pass this region.

Temporary increasing the uncertainty threshold $\alpha$ allows robots to pass regions with few or no landmarks. However, the threshold $\alpha$ should be reduced as soon as possible to avoid large error in the exploration process afterwards. In our method, $\alpha$ is reduced whenever the robot pose uncertainty decreases and the landmark density in the neighboring region of the robot is sufficiently high.

### 3.5 Limited communication range

In many methods, including [12], it is assumed that the robots can always establish a wireless network to communicate with one another. However, the communication range is limited in real situations. One way to deal with the limited communication problem is to keep robots close to one another. Simulation results in [12] revealed that this strategy leads to a longer exploration time if the communication range is not sufficiently large. Another way to deal with this problem is to allow robots to move out of the communication range. However, when a robot does not know where other robots are, it can unawarely follow others, leading to an overlapping exploration.

To overcome the above problems, in the proposed method robots are allowed to temporary move out of the communication range and then rendezvous with one another after a specific time. This amount of time should depend on the communication range, robot’s sensor range, robot’s velocity and even the environment. The determination of the rendezvous point is delineated in the following. As far as rendezvous is concerned, a landmark with low position uncertainty can serve as a
target point. The central location of the robot team is also considered as a target point. Similar to the exploration and relocalization phases, the target point for rendezvous is determined by optimizing an objective function that is related to the cost of navigation $C_{N3}$, utility of localizability $U_{L3}$, and distance to the nearest exploration point $D$ (excluding current destinations of all robots). The objective function of the $i$-th target is given by

$$J_{\text{rendezvous}} = w_{L3}U_{L3} - w_{N3}C_{N3} - w_{D3}D$$

(8)

where $w_{L3}$, $w_{N3}$, and $w_{D3}$ are weighting coefficients. The utility of localizability $U_{L3}$ is used as in the exploration and relocalization phases to distinguish between target points with different localization qualities. The distance $D$ is similar to that in Eq. (7). The cost of navigation $C_{N3}$ is given by the total travelling distance from the positions of all robots to the target. In the present study, the cost is simply the sum of Euclidian distances between all robots and the target point. Once the robot is ready to move out of the communication range, the objective function $J_{\text{rendezvous}}$ in Eq. (8) as a function of all candidate target points is optimized. Once the target point is determined, all robots store this location as well as the current time and begin to explore beyond the communication range within a preset time.

### 4. SIMULATION RESULTS

In the simulations, two robots coordinate with each other to completely explore an environment. Control signals are applied to the robots every 0.025 s and range and bearing measurements are taken every 0.2 s. The mean velocity of the robots is 3 m/s and the sensor range is 20 m. The destination is recomputed whenever a robot reached its current destination or a robot has moved 3 m.

#### 4.1 Adaptive threshold for robot pose uncertainties

In this simulation, the environment contains 74 landmarks located in an area of $250 \times 80$ m$^2$ as depicted in Fig. 2. Three different cases as listed in Table 2 are indeed considered. In the simulation analysis, 10 independent runs for each case are performed.

<table>
<thead>
<tr>
<th>Case</th>
<th>Allow to increase threshold</th>
<th>Allow to reduce threshold</th>
</tr>
</thead>
<tbody>
<tr>
<td>A [12]</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>B</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>C (proposed)</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Table 2: Cases under consideration.

Table 3 delineates the RMS position errors of the two robots and the error in landmark estimation of these three cases. The number of steps which is related to the exploration time is also provided. It is observed that Case A has the longest exploration time because Robot 1 is stuck in Region 1 thus the environment is almost explored by only Robot 2. Case B has the biggest error because the uncertainty threshold is increased in Region 1 and kept unchanged afterwards thus robots do not exploit the relocalization phase in Region 3, leading to large errors in this region. In Case C, the threshold is temporarily increased in Region 1 in order to allow Robot 1 to pass this region. After meeting Robot 2 in Region 2, the threshold is reduced allowing robots to exploit the relocalization phase in Region 3, leading to the more accurate map and trajectories in this region. As a result, Case C has as low errors as Case A while the exploration time is very close to that in Case B.

#### 4.2 Limited communication range

In this simulation, the environment contains 59 landmarks located in an area of $150 \times 150$ m$^2$ as depicted in Fig. 3. The proposed approach (case D) is compared with the method in [12] in which robots are allowed to move out of the communication range but not to rendezvous (case E). A typical result of case D is depicted in Fig. 3. Meeting points during the exploration process are in turn marked as M1, M2 and M3. 10 independent runs for each case are performed. Tables 4 compare the RMS robot position errors and landmark error for these two cases.

It can be seen that the errors in Case D are comparable with those in Case E while the exploration time in Case D is only 80% of that in Case E.
Table 4 RMS errors and number of steps.

<table>
<thead>
<tr>
<th>Case</th>
<th>Robot error (m)</th>
<th>Landmark error (m)</th>
<th>No. of steps</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>0.684</td>
<td>0.741</td>
<td>0.246</td>
</tr>
<tr>
<td>E</td>
<td>1.024</td>
<td>0.697</td>
<td>0.131</td>
</tr>
</tbody>
</table>

5. CONCLUSIONS

An improved version of the active SLAM algorithm for multi-robot exploration is proposed. Based on the framework in [12], the paper makes two important improvements. Firstly, an adaptive strategy is employed to automatically adjust the threshold of the robot pose uncertainty constraints in order to prevent the robots from oscillating between the exploration and relocalization phases. Secondly, rendezvous technique is utilized to deal with the limited communication range by allowing robots to temporarily move out of the communication range and rejoin the group later. Simulation results show that the improved approach outperforms the original one.

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