Blind Channel Estimation with Periodicity for OFDM Systems without Cyclic Prefix

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Abstract—Generation of the signal and noise subspaces is a critical problem in subspace-based algorithms for orthogonal frequency division multiplexing (OFDM) systems. Some special characteristics, such as virtual carriers (VCs), real symbols, and/or cyclic prefix (CP), can be exploited to construct the required noise subspace in conventional subspace approaches. In this paper, a blind channel estimation algorithm with periodicity property is proposed for OFDM systems without CP. Using the time-domain periodicity, which can be obtained by inserting zeros at some positions of frequency-domain OFDM symbols, a method for constructing the noise subspace is developed based on the proposed signal model. Simulation results show that the proposed blind channel estimation method has better normalized mean-squared error (NMSE) performance than that of a conventional approach with VCs.

I. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) [1] is commonly used in wireless communication systems due to their higher bandwidth efficiency and data rate compared to that of conventional communication systems. In OFDM systems, several small bands can be obtained from the wideband spectrum, and thus the fading effects on each subcarrier can be regarded as frequency non-selective fading. Therefore, a simple one-tap frequency-domain equalizer can be used to compensate for the fading effects in the design of OFDM receivers.

Accurate channel estimation with coherent detection is an important issue in receiver design since it has a 3dB signal-to-noise ratio (SNR) gain over differential detection. Channel estimation approaches can be divided into data-aided and blind approaches. Data-aided algorithms [2], [3] require extra known data, such as pilot symbols, for channel estimation. On the other hand, the known information is not required in blind channel estimation algorithms [4]-[10]. The subspace approach in blind channel estimation was first proposed in [4]. It applies singular value decomposition (SVD) to find the singular vectors of the noise subspace. After these singular vectors are obtained, a relation between the noise singular vector and the channel impulse response (CIR) can be established to estimate the CIR. Subspace-based channel estimation [5]-[7] has been widely discussed in conventional OFDM systems using virtual carriers (VCs) or cyclic prefix (CP). For multiple-input multiple-output (MIMO) applications [8], [9], this approach can be used to estimate the CIR efficiently. A deterministic method called cross-relation (CR) [10] has also been proposed for blind channel estimation. The CR approach constructs the relation between received data symbols and the channel matrix. However, this relation only holds in communication systems with multiple receive antennas and does not exist in SISO systems.

A subspace-based method for CP-free OFDM systems which applies the VCs, referred to as the Li method hereafter, was presented in [6]. In this approach, the noise subspace can be constructed for SISO-OFDM systems without oversampling techniques or multiple antennas. However, the dimension of the noise subspace has to be increased with many consecutive OFDM symbols to obtain a good channel estimate. Therefore, the method has high computational complexity because a large matrix is decomposed. In the present study, a subspace-based blind channel estimation algorithm which uses the periodicity of time-domain OFDM symbols is proposed. To make the time-domain OFDM symbols periodic, some frequency-domain data symbols are set to zero at specific positions. Then, a new signal model with time-domain periodicity is constructed to generate the noise subspace for subspace-based channel estimation. Compared to conventional blind channel estimation with VCs, the proposed method has better normalized mean-squared error (NMSE) performance.

The rest of this paper is organized as follows. Section II presents the signal model of CP-free OFDM systems. In Section III, the proposed subspace-based blind channel estimation using the time-domain periodicity is described. Finally, simulation results and conclusions are given in Section IV and Section V, respectively.

II. SIGNAL MODEL

In this paper, matrices and column vectors are represented by boldfaced uppercase and lowercase letters, respectively. (·)\text{T} and (·)\text{H} denote the operators of transpose and transpose-
conjugate, respectively. The symbol $\|\|$ denotes the Frobenius norm of the matrix argument. Consider a signal model for CP-free OFDM systems with one transmit antenna and one receive antenna. The system model of a single-input single-output (SISO) OFDM system without CP is shown in Fig. 1. The $k^{th}$ frequency-domain transmitted OFDM symbol with $N$ total subcarriers can be written as

$$d_k = [d_{k,0} \ d_{k,1} \ldots \ d_{k,N-1}]^T$$  \hspace{1cm} (1)

where $d_{kj}$ is chosen from a complex modulated signal constellation and the symbol $i$ is the index of the subcarrier. To obtain the time-domain OFDM symbol, the frequency-domain data symbol passes through an inverse discrete Fourier transform (IDFT) or inverse fast Fourier transform (IFFT) block. The $k^{th}$ time-domain OFDM symbol is

$$x_k = Td_k = [x_{k,0} \ x_{k,1} \ldots \ x_{k,N-1}]^T$$  \hspace{1cm} (2)

where $T$ represents an $N \times N$ IDFT matrix. The CIR with maximum channel delay spread length $L$ is defined as

$$h = [h_0 \ h_1 \ldots h_L]^T.$$  \hspace{1cm} (3)

Since the synchronization between the transmitter and the receiver is assumed to be perfectly performed before channel estimation, the carrier frequency offset (CFO) and timing offset are not considered in our signal model. The received $(N-L) \times 1$ time-domain OFDM symbols without inter-symbol interference (ISI) corresponding to the $k^{th}$ transmitted OFDM symbol $x_k$ can be represented as

$$y_k = H x_k + n_k$$  \hspace{1cm} (4)

where

$$H = \begin{bmatrix} h_L & h_{L-1} & \ldots & h_0 & 0 & \ldots & 0 \\ 0 & h_L & h_{L-1} & \ldots & h_0 & \ldots & \vdots \\ \vdots & \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \ldots & 0 & h_L & h_{L-1} & \ldots & h_0 \end{bmatrix}$$

is a Toeplitz matrix with dimensions $(N-L) \times N$ and $n_k$ is an additive white Gaussian noise (AWGN) vector with dimensions $(N-L) \times 1$. Define that the first $L$ and last $L$ symbols of the $k^{th}$ transmitted time-domain OFDM symbol $x_k$ are $x_{k,U}$ and $x_{k,L}$, respectively. Then, the $k^{th}$ $L \times 1$ received symbol $y_{ISI,k}$ in time domain with ISI can be written as

$$y_{ISI,k} = H_L x_{k,U} + H_U x_{k-1,L} + n_{ISI,k}$$  \hspace{1cm} (5)

where

$$H_L = \begin{bmatrix} h_0 & 0 & \vdots \\ \vdots & \ddots & \vdots \\ h_{L-1} & \ldots & h_0 \end{bmatrix}$$ \hspace{1cm} and \hspace{1cm} $$H_U = \begin{bmatrix} h_L & \cdots & h_1 \\ 0 & \cdots & \vdots \end{bmatrix}$$

are $L \times L$ matrices and $n_{ISI,k}$ is the AWGN vector with dimensions $L \times 1$.

Fig. 1. OFDM system model without CP.

III. PROPOSED CHANNEL ESTIMATION ALGORITHM

A. Channel Equations of the Proposed Algorithm

To obtain the periodicity of transmitted time-domain OFDM symbols, $N/2$ zeros are inserted into the frequency-domain OFDM symbols $d_k$. Therefore, the $k^{th}$ frequency-domain transmitted OFDM symbol becomes

$$d_k = [d_{k,0} \ 0 \ d_{k,1} \ 0 \ldots d_{k,N/2-1} \ 0]^T.$$  \hspace{1cm} (6)

According to the IFFT/FFT property, the $k^{th}$ time-domain OFDM symbol can be written as

$$x_k = [x_{k,0} \ x_{k,1} \ldots x_{k,N/2-1} \ x_{k,0} \ x_{k,1} \ldots x_{k,N/2-1}]^T.$$  \hspace{1cm} (7)

where the upper part and lower part of $x_k$ are the same. Define $x_{k,0} = [x_{k,0} \ x_{k,1} \ldots x_{k,N/2-1}]^T$. Then, $x_k$ becomes

$$x_k = [x_{k,0} \ x_{k,0}]^T.$$  \hspace{1cm} (8)

From (8), we can rewrite (4) into the following form

$$y_k = H x_k + n_k = [H_1 \ H_2] \begin{bmatrix} x_{k,0} \\ x_{k,0} \end{bmatrix} + n_k$$  \hspace{1cm} (9)

where $H_1$ and $H_2$ are $(N-L) \times N/2$ matrices. Since the upper part and lower part of $x_k$ are the same, (9) becomes

$$y_k = H c x_{k,0} + n_k$$  \hspace{1cm} (10)
where $\mathbf{H}_c$ is expressed by

$$\mathbf{H}_c = \mathbf{H}_1 + \mathbf{H}_2. \quad \text{(11)}$$

Consider two consecutive OFDM symbols. We can obtain a $(2N-L)\times 1$ composite block, which is defined as

$$\mathbf{y}_k = \begin{bmatrix} \mathbf{y}_{k-1}^T & \mathbf{y}_{k_L}^T & \mathbf{y}_{k}^T \end{bmatrix}^T. \quad \text{(12)}$$

The composite block can be rewritten as

$$\mathbf{y}_k = \mathbf{H}\mathbf{x}_k + \mathbf{n}_k \quad \text{(13)}$$

where

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_c & 0 \\ 0 \mathbf{H}_c & 0 \\ 0 & \mathbf{H}_c \end{bmatrix} \quad \text{and} \quad \mathbf{x}_k = \begin{bmatrix} \mathbf{x}_{k-1,0} \\ \mathbf{x}_{k,0} \end{bmatrix}.$$

Note that the dimensions of $\mathbf{H}_c$, $\mathbf{x}_k$, and $\mathbf{n}_k$ are $(2N-L)\times N$, $N\times 1$, and $(2N-L)\times 1$, respectively. Consider $I$ received OFDM symbols, (13) can be rewritten as

$$\mathbf{Y} = \mathbf{H}\mathbf{X} + \mathbf{N} \quad \text{(14)}$$

where the dimensions of $\mathbf{Y}$, $\mathbf{X}$, and $\mathbf{N}$ are $(2N-L)\times (I-1)$, $N\times (I-1)$, and $(2N-L)\times (I-1)$, respectively. According to the signal model shown in (14), the observation dimensions and the signal dimensions become $(2N-L)$ and $N$, respectively. Therefore, the noise dimensions are $(N-L)$ according to the subspace approach. In the next subsection, the blind channel estimation based on the subspace algorithm is shown.

### B. Subspace-Based Channel Estimator

The annihilators of $\mathbf{Y}$ can be obtained by taking the SVD of $\mathbf{Y}$. Assume that $\mathbf{W}_n(i)$ ($i = 0, 1, \ldots, N-L-1$) are annihilators. Then, the following equation holds under noiseless environments

$$\mathbf{W}_n(i)^H \mathbf{Y} = \mathbf{0}^T. \quad \text{(15)}$$

The annihilators of $\mathbf{Y}$ are also the annihilators of $\mathbf{H}$ only if the signal matrix $\mathbf{X}$ is full row rank. Assuming that $\mathbf{W}_n(i)$ is the annihilator of $\mathbf{H}$, we can obtain the following relation

$$\mathbf{W}_n(i)^H \mathbf{H} = \mathbf{0}^T. \quad \text{(16)}$$

Define $\mathbf{h} = \begin{bmatrix} h_{L-1} & \cdots & h_0 \end{bmatrix}^T$. Then (16) can be rewritten as

$$\mathbf{W}_n(i)^T \mathbf{H} = \mathbf{n}^T \mathbf{W}_i \quad \text{(17)}$$

Now, if we define

$$\mathbf{W} = \begin{bmatrix} \mathbf{W}_0 & \mathbf{W}_1 & \cdots & \mathbf{W}_{N-L-1} \end{bmatrix}^T \quad \text{(18)}$$

we can obtain $\mathbf{h}^T \mathbf{W} = \mathbf{0}$. If noise is present, the estimated annihilator $\mathbf{W}_n(i)$ can be obtained by taking the SVD of $\mathbf{Y}$.

Estimated annihilators $\mathbf{W}_n(i)$ come from the $(N-L)$ singular vectors associated with the $(N-L)$ smallest singular values. Therefore, the CIR can be estimated by

$$\hat{\mathbf{h}} = \arg \min \left\| \mathbf{W}_n(i)^H \mathbf{H} \right\|^2 \quad \text{(19)}$$

where

$$\left\| \mathbf{W}_n(i)^H \mathbf{H} \right\|^2 = \mathbf{W}_n(i)^H \mathbf{H} \mathbf{H}^H \mathbf{W}_n(i) = \mathbf{h}^T \mathbf{W}_i^T \mathbf{W}_W \hat{\mathbf{h}} + \mathbf{n}^T \mathbf{W}_i^T \mathbf{W}_W \hat{\mathbf{h}}.$$

If we define $\mathbf{h} = (\hat{\mathbf{h}})^*$, the channel estimator becomes

$$\hat{\mathbf{h}} = \arg \min \left\| \mathbf{h}^H \mathbf{W} \mathbf{W}^H \mathbf{h} \right\| \quad \text{(20)}$$

### C. Discussion

The computational complexity of the proposed algorithm is based on the SVD of the matrix $\mathbf{Y}$, whose dimensions are $(2N-L)\times (I-1)$; thus, the complexity is proportional to $O((2N-L)^3)$. Since the dimensions of the matrix used in the SVD operation in the Li method are $(JN-L)\times K$, where $J$ is the number of used OFDM symbols for each column and $K$ is equal to $L-(I-1)$, the complexity of the Li method is proportional to $O((JN-L)^3)$. To improve performance in the Li method, $J$ is usually equal to or larger than two. Therefore, we can conclude that the computational complexity of the proposed method is lower than that of the Li method.

### IV. Simulation Results

Several Monte Carlo simulations were conducted to verify the performance of the proposed approach. In the simulations, the timing and frequency synchronization is assumed to be performed before channel estimation. A CP-free OFDM system with one transmit antenna and one receive antenna is considered. The length of the OFDM symbol is chosen as $N=16$. The data symbols are chosen from the BPSK or QPSK constellation. The maximum number of paths of the CIR is assumed to be $L+1$ ($L=4$). The following exponential power delay profile is applied for each channel path [3].

$$E[|h_l|^2] = e^{-l/10}, \quad l = 0, 1, \ldots, L \quad \text{(21)}$$

The phase of each channel path is uniformly distributed over $[0, 2\pi)$. The normalized mean-squared error (NMSE) is used to examine the performance of the channel estimation algorithms. The NMSE is defined as

$$\text{NMSE} = \left(\frac{1}{M} \left\| \mathbf{h}_m - \mathbf{h} \right\|^2 \right) \left( \frac{1}{M} \sum_{m=1}^{M} \left\| \mathbf{h}_m - \mathbf{h} \right\|^2 \right) \quad \text{(22)}$$
where $M$ denotes the number of computer runs, the index $m$ represents the $m$th Monte Carlo run, and the parameter $c$ is the phase ambiguity, which can be solved by one extra pilot symbol. Moreover, $\hat{h}_m$ is the $m$th estimated channel and $h$ is the true channel impulse response. Since periodicity is applied in the proposed approach, only $N/2$ valid complex data symbols are received in an OFDM symbol compared to these in conventional OFDM systems. In order to make the comparison between the Li method and the proposed approach fair, $N/2=8$ data symbols are used in the Li method; thus, the number of VCs is also $N/2=8$. Note that using a larger number of VCs in the Li method enlarges the dimensions of the noise subspace, making the channel estimate more accurate.

A comparison of the NMSE performance between the Li method and the proposed approach under BPSK modulation is shown in Fig. 2. When $I$ is 17, 30, 50, or 70, the NMSE is a decreasing function of SNR for both the Li method and the proposed approach. However, the NMSEs of the proposed method are smaller than those of the Li method. Thus, the proposed algorithm outperforms the Li method in NMSE. Fig. 3 shows the NMSE performance comparison under QPSK modulation between the Li method and the proposed algorithm. Similar to the results shown in Fig. 2, the proposed algorithm outperforms the Li method for $I=17, 30, 50, or 70$.

V. CONCLUSIONS

A blind channel estimation algorithm using the subspace method for OFDM systems without CP was presented in this paper. By inserting zeros in some positions of the transmitted OFDM symbol in the frequency domain, the time-domain periodicity of the transmitted signal can be obtained. The periodicity is applied to generate the noise subspace for channel estimation. Simulation results show that the proposed subspace-based approach outperforms a conventional method with VCs under exponential-decay static channels. The proposed method is also suitable for various modulation schemes such as BPSK and QPSK.

REFERENCES