Reconstruction Synthesis of the Lost Subsystem for the Planetary Motions of Antikythera Mechanism

Authors: Hong-Sen Yan & Jian-Liang Lin

Speaker: Jian-Liang Lin

Department of Mechanical Engineering
National Cheng Kung University
Tainan, Taiwan
Outline

- Introduction
- Overall Contribution
- Historical Background
- Kinematic Analysis
- Feasible Topological Structures
- Process of Reconstruction Synthesis
- Conclusions
Introduction

Display the motions of heavenly bodies
Do the calculations of different calendars
Predict the time of eclipses

Creative Machine Design E&R Lab, National Cheng Kung University, TAIWAN
Systematic Approach of Reconstruction Synthesis

1. Historical Archives
2. Design specifications
   - Number synthesis
   - Topological characteristics
   - Generalized Kinematic Chains
     - Specialization
     - Design constraints
     - Specialized Chains
6. Ancient science and technology

Creative Machine Design E&R Lab, National Cheng Kung University, TAIWAN
Overall Contribution

- Two examples of the lost planetary motions subsystem are derived respectively with one and three design concepts.
Historical Background

Venus indicator
L: 50 teeth
D: ≈65 teeth, rotates once per Earth year

Fixed wheel with 80 teeth

Existing reconstruction designs from 2000 to date

Creative Machine Design E&R Lab, National Cheng Kung University, TAIWAN
Historical Background

- The feasible astronomical theory
  Apollonius developed the epicycles model to account for the motions of five planets. And he successfully produced the retrograde motion of planets.
Kinematic Analysis

- Apollonius’ epicycles model
  The planet rotates on an epicycle whose center rotates on the deferent with the same direction.

\[ \omega = \omega_d + \omega_{de} \]

\[ \omega = \omega_d + \left( \frac{r_e}{r_d} \cos(\theta_e - \theta_{de}) \right) \left( \frac{1 + \frac{r_e}{r_d} \cos(\theta_e - \theta_{de})}{\omega_e} \right) \]

Anomaly Function
Feasible Topological Structures

Q1: Input Identification of Lost Subsystem...

Q2: The numbers of links and joints............

Q3: The position of pin-in-slot joint............

Creative Machine Design E&R Lab, National Cheng Kung University, TAIWAN
Q1: Input Identification

Three feasible input conditions of lost planetary motions subsystem were supposed.
Q2: Numbers of Links and Joints

The lost planetary subsystem with \( N_L \) members, \( N_{JR} \) revolute pairs, \( N_{JA} \) cam pairs and \( N_{JG} \) gear pairs has one degree of freedom.

Based on the mobility analysis:
\[
2N_{JR} + N_{JG} + N_{JA} - 3N_L + 4 = 0 \quad \ldots \quad (1)
\]

For a gear train with \( N_J \) joints:
\[
N_{JR} + N_{JG} + N_{JA} - N_J = 0 \quad \ldots \quad (2)
\]

Each link in a geared kinematic chain must have one revolute pair:
\[
N_{JR} - N_L + 1 = 0 \quad \ldots \quad (3)
\]
Q2: Numbers of Links and Joints

By solving above equations, the relations between the joints and links could be derived as:

**All Joint number**

\[ N_J = 2N_L - 3 \]

**Revolute Joint number**

\[ N_{JR} = N_L - 1 \]

**Gear Joint number**

\[ N_{JG} = N_L - 3 \]

**Pin-in-slot Joint number**

\[ N_{JA} = 1 \]

- \( N_{JG} = 1 \)  
  *Four-bar mechanism with five joints*

- \( N_{JG} = 2 \)  
  *Five-bar mechanism with seven joints*
Q3: Positions of Pin-in-slot Joint

For a planetary gear train with \( n \) planet gears and one pin-in-slot joint:

**The pin-in-slot joint is incident to the planet gear and the output link.**

\[
\frac{T_s}{T_{pn}} = \frac{\omega_e}{\omega_d} = \frac{\text{the rate of epicycle}}{\text{the rate of deferent}}
\]

**The pin-in-slot joint is incident to two planet gears.**

\[
(-1)^n \frac{T_s T_{p_k}}{T_{p_{(k-1)}}} = \frac{\omega_d}{\omega_e} = \frac{\text{the rate of deferent}}{\text{the rate of epicycle}}
\]
Design Specifications of Each Type

**Type 1**
1. It is a planetary gear train.
2. It is a **four-bar mechanism with five joints and one degree of freedom.**
3. The joints include three revolute joints, one external gear joint.
4. The types of mechanical components are gears and links.

**Type 2**
1. It is a planetary gear train.
2. It is a **five-bar mechanism with seven joints and one degree of freedom.**
3. The joints include four revolute joints, two external gear joints and one pin-in-slot joint.
4. The types of mechanical components are gears and links.
Specialization Process of Type 1

It must have at least a ground link (member 1, K_F), an input link (member 2, K_I), an output link (member 3, K_O), and a transmission link (member 4, K_{T1}).

STEP 1: Ground link
STEP 2: Input link
STEP 3: Output link
STEP 4: Transmission link
STEP 5: Pin-in-slot joint
STEP 6: Revolute joints & Gear joints
Feasible Design Concept of Type 1

One feasible design concept for the inferior and superior planets

The inner displays the pointer of date
The outer displays the pointer of the planet

3
1 4
2
b1
axis b
Specialization Process of Type 2

1. **Ground link**
   - Step 1
   - Step 2: Not feasible
   - Step 3
   - Step 4
   - Step 5

2. **Input link**
   - Step 2: Not feasible
   - Step 3
   - Step 4
   - Step 5

3. **Output link**
   - Step 3
   - Step 4: Not feasible
   - Step 5

4. **Transmission link**
   - Step 4: Not feasible
   - Step 5

5. **Pin-in-slot joint**
   - Step 5

6. **Revolute joints & Gear joints**
   - Step 6: Not feasible
Feasible Design Concepts of Type 2

The outer displays the pointer of the planet
The inner displays the pointer of date

The inner displays the pointer of date
The outer displays the pointer of the planet

Creative Machine Design E&R Lab, National Cheng Kung University, TAIWAN
Conclusions

This work synthesizes all feasible reconstruction designs of the planetary subsystem through a systematic design procedure.
Conclusions

- The kinematic analysis of epicycles model and the discussions of feasible topological structures were presented.
Contribution

Two examples of the lost planetary motions subsystem are derived respectively with one and three design concepts.

[Diagram of planetary motion subsystems]
Acknowledgement

The authors are grateful to the National Science Council (TAIWAN, ROC) under Grant NSC 99-2221-E-006-253 for the financial support of this work.
Thanks for your attentions!