Discussion of “Generalized Method for Three-Dimensional Slope Stability Analysis” by Ching-Chuan Huang, Cheng-Chen Tsai, and Yu-Hong Chen

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Introduction

The authors are to be congratulated for their new method satisfying two-directional force and moment equilibrium which is potentially useful as a practical approach for three-dimensional (3D) stability analysis of geometrically and geologically complicated slopes. The following comments are offered and a recommendation necessary to demonstrate performance of a generalized 3D model for slope stability computations is then presented.

Section 1

In the past 25 years, almost all slice methods commonly used for two-dimensional (2D) analysis of slope stability have been extended into 3D as column methods. One of the typically different characteristics of a 3D problem from that of 2D analysis is that the direction of movement of potential sliding masses is unknown. According to the slip mechanism used for making the problem determinate, it is convenient to classify existing limit equilibrium-based methods for 3D slope stability into the following two categories:

1. Methods in which the entire sliding mass is assumed to move in a single direction (i.e., the direction of sliding is uniform for the whole soil mass over a potential slip surface). Almost all existing methods of columns are based on such an assumption; some can only be used in the analysis of slopes with a symmetrical failure surface (Xing 1988), and others are valid for slopes with any geometry of potential slip surface (Ugai 1988; Hungr et al. 1989; Lam and Fredlund 1993).

2. Methods in which the direction of sliding is obtained as a part of the analytical solution rather than being assumed in advance. As a result, the computed direction of sliding (i.e., direction of mobilized resultant of shear force on the slip surface) may vary from place to place over the potential slip surface. Variational limit equilibrium approaches (Leshchinsky and Huang 1992) consider variable direction of sliding and satisfy all limit equilibrium equations, but they are only applicable to symmetrical 3D slope-stability problems. The two-directional force and moment equilibrium method posed by the authors falls, of course, into this category, and it has the potential to analyze asymmetrical slopes with arbitrary slip surface shape.

Section 2

The slip mechanism of a single direction of movement for the entire sliding mass leads to an assumption that the shear force acting on the slip surface has no components in the direction transverse to the main sliding [i.e., the mobilized shear force acting on the base of the slip surface is assumed to be parallel to the symmetrical plane (the x-z plane)]. Although such an assumption may oversimplify the actual problems in complicated conditions, it has been shown that use of a uniform movement direction for the entire sliding mass is appropriate for the analysis of symmetrical 3D slopes. In such cases, symmetrical critical slip surfaces take place, and hence, the distribution of the resultant shear force acting on the slip surface should also be symmetrical about the x-z plane. This means that the transverse-direction components of the shear force at two symmetrical points on the slip surface have the same magnitude but act in the opposite direction. Consequently, when the limit equilibrium equations of the whole sliding mass are considered, their effects in the transverse direction of sliding will be cancelled out. In other words, force equilibrium and moment equilibrium in the transverse direction are automatically satisfied due to symmetry of the problem. For 3D sliding masses, which can approximately be regarded as being symmetrical about the x-z plane, therefore, little inaccuracy in the stability analysis due to the single sliding-direction assumption may occur. The results provided by the authors for symmetrical slopes have also shown that the difference between factors of safety computed on specified 3D slip surfaces using existing column methods and their two-directional force and moment equilibrium approach was rather small. Although the comparative study of the authors was restricted only to specified 3D slip surfaces in symmetrical slopes, a meaningful and more persuasive comparison should be made under most critical conditions (i.e., comparing critical slip surfaces and values of minimum factors of safety). This motivated the discussers to locate the 3D critical slip surfaces for typical symmetrical slopes by incorporating the search scheme presented by Yamagami and Jiang (1997) into a 3D Spencer method (Ugai 1988) associated with a single sliding direction and to compare those obtained from the variational analysis (Leshchinsky and Huang 1992) that considered variable direction of sliding along the potential slip surface. As a result, both the 3D critical slip surfaces and associated minimum factors of safety of the 3D Spencer method agree well with those by the variational computations (Jiang and Yamagami 2002), indicating the effectiveness of one-directional force and moment equilibrium methods with a single sliding-direction assumption for symmetrical slope-stability problems.

The authors state in their conclusions that “This method was validated using 2D and 3D failure surfaces reported in the literature.” As all the failure surfaces reported in the literature are symmetrical about the x-z plane, the two-directional force and moment equilibrium method was not validated well. In other
words, the importance of satisfying the force and moment equilibrium in the transverse direction cannot be verified by comparison only with symmetrical sliding masses. As unbalanced force and/or moment in the transverse direction of sliding may result in errors in factors of safety calculated for nonsymmetrical problems (Hungr et al. 1989; Huang et al. 2000), it will be interesting, and also necessary, to apply the current approach to asymmetrical failure surfaces. This is to examine how much potential error is due to the single sliding-direction assumption and to what extent this error can be eliminated by introduction of force and moment equilibrium in the transverse direction.

**Section 3**

When conventional column methods are used for the analysis of asymmetrical 3D slopes, a sliding direction must be assumed and an individual coordinate system is necessary for the analysis to follow the assumed direction of sliding (the \( x \)-axis is usually chosen to be parallel to the sliding direction). It has been shown that the results of stability computation may be significantly influenced by the assumed direction of sliding (Stark and Eid 1998), and the main sliding direction is determined as a result of searching for the most critical slip surface (Jiang et al. 2003). In the authors’ method, formulation of the factor of safety was carried out in a unified coordinate system, and an assumption of the sliding direction was not necessary. However, there is no mathematical proof that the calculated direction of sliding as well as the factor of safety does not vary with change of the chosen \( x \)-axis (or \( y \)-axis) of the coordinate system. One can select an arbitrary direction in the \( x-y \) plane as the \( x \)-axis (or \( y \)-axis) for the analysis of an asymmetrical slope, thus, it is physically important to check if the results of the analysis are affected by the selected direction of the \( x \)-axis (or \( y \)-axis). This was verified only for a given symmetrical slip surface within a truncated cone using different \( \psi \) angles (\( \psi \)=the angle between the \( x \)-axis and the projection of the symmetrical plane on the \( x-y \) plane), as shown in Fig. 18 and Table 6 in the original paper. As mentioned previously, however, simply addressing the numerical calculation for symmetrical slip surfaces without examining performance of the authors’ model for nonsymmetrical problems may not provide a generalized method for 3D slope-stability analysis. The discussers believe that the authors should analyze an asymmetrical slope failure to show insignificant variation in the safety factors and sliding directions calculated using the generalized model with the \( x \)-axis of the coordinate systems chosen at different directions in the \( x-y \) plane.

**Section 4**

In connection with the issues described previously, and considering that the balanced force and moment equilibrium in the transverse direction of sliding is potentially significant for asymmetrical problems, it is recommended that a typical nonsymmetrical slip surface should be analyzed so as (1) to look into to what extent the possible error due to ignoring force and moment equilibrium in the transverse direction can be eliminated by the authors’ method, and more importantly (2) to demonstrate that the coordinate systems with an arbitrarily chosen \( x \)-axis in a horizontal \((x-y)\) plane do not result in a significant change in results of analysis. Actual failed slopes have a well-defined failure surface, for which the minimum 3D factor of safety is known (equal to unity) and the direction of movement of the entire sliding mass can also be obtained through field observation or site investigation. Therefore, it is most preferable to apply the current method in an actual slope failure with a well-defined 3D slip surface so as to validate its performance.

**References**


**Closure to “Generalized Method for Three-Dimensional Slope Stability Analysis” by Ching-Chuan Huang, Cheng-Chen Tsai, and Yu-Hong Chen**


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**Introduction**

The writers appreciate some valuable comments from the discussers. However, the discussers must distinguish between the focus
and the possible future works. Comments from the discussers can be summarized into the following:

1. A meaningful and more persuasive comparison should be made under most critical conditions rather than specified three-dimensional slip surfaces,

2. The two-directional force and moment equilibrium method was not validated because the failure surfaces used for comparisons are all symmetrical ones. A typical asymmetrical slip surface should be analyzed to investigate possible error associated with ignoring force and moment equilibrium in the transverse direction, and

3. No mathematical proof for the consistency of safety factors with the change of coordinate system.

For Comment 1, the discussers failed to recognize that comparisons made for specified failure surfaces are more fundamentally important than those made for the most critical conditions. This is obvious because a less accurate method may provide lower safety factors resulting in misleading conclusions.

For Comment 2, possible error for the calculated safety factors when ignoring force and moment equilibrium can be significantly large. This has been demonstrated by Huang and Tsai (2000).

Further studies are necessary for quantitative and generalized conclusions on this topic.

For Comment 3, the analytical results shown in Figs. 14 and 16 of the original paper provide numerical, rather than mathematical, proof of the consistency on safety factors calculated with the changing coordinate system. No mathematical proof is required when using this trial and error based–limit equilibrium method. When we examine the relative position between the generalized coordinate system (x and y axis) and all failure surfaces for the truncated cone (Fig. 14), it is clear that all failure surfaces, except those under \( \Psi = 0^\circ, 90^\circ, 180^\circ, \) and \( 360^\circ \) conditions, are under asymmetrical and rotated conditions with respect to the x and/or y axis. The analytical result shown in Fig. 16 has already demonstrated the capability of providing consistent safety factors for asymmetrical failure surfaces regardless of the change of coordinate system.

References