Simulation study of relativistic dynamics of MeV alpha particles in magnetized plasmas for explaining an experimental anomaly

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In a Test Fusion Tokamak Reactor [R. J. Hawryluk et al., Phys. Plasmas 5, 1577 (1998)] experiment, the measured energy spectrum of the deeply trapped alpha particles is found to be 1 MeV too broad to be explained by classical collisions and the peak energy similarly off by 450 keV. The relativistic effect is proposed as an explanation. Here, we report high-resolution Monte Carlo (MC) and particle-in-cell (PIC) simulation studies in detail, under the assumption of a uniform magnetic field, for the identification of the cause of the observed anomaly. The 3.5 MeV alpha particles produced by thermonuclear fusion reaction are broadened due to Doppler effect. The relativistic alpha particle dynamics are followed with the PIC code. The relativistic ion cyclotron instability grows to saturation on a time scale ($10^{-5}$ s) much shorter than the experimental time scale of 0.1 s. The MC code is then used to follow, in real time, the collisional slowing down of the gyrobroadened alphas, including the effect of the time delay in diagnostic pellet releasing and flight. Relativistic gyrobroadening is shown to be crucial in shaping the birth and slowed-down spectra. The resultant alpha particle energy spectrum fits well with that of the measurement, with a reduced chi square of unity. © 2005 American Institute of Physics. [DOI: 10.1063/1.2148912]

I. INTRODUCTION

The physics of dynamical behavior of fast ions is of fundamental importance in plasma studies with applications in fusion,3–30 space,31,32 and astrophysics33 researches. It has been shown2–7 that the relativistic ion cyclotron instabilities can significantly affect the fast ion dynamics. The characteristics of the instabilities and the consequences on the dynamics are extremely sensitive to the mass deficit of fast ions in relation with that of slow ions.

In burning fusion reactors, the 3.5 MeV alpha particles produced by the reaction of deuterons and tritons are the only direct source for heating the plasmas to maintain the self-burning,8–11 and thus their stability and dynamics are the critical issues12–15. To explore the possible relativistic effect during the slowing down process of the alphas in magnetized plasmas, an experiment16,17 at Princeton’s Test Fusion Tokamak Reactor (TFTR) measures the alpha particle energy spectra at the earlier time. The conclusion was that the alpha particles are classically slowed down. However, more recently, Chen7 has revealed that there is indeed an experimental anomaly of the measured alpha energy spectrum, after carefully examining the plasma parameters and alpha particle dynamics. The width of the alpha particle energy spectrum is 1 MeV broader than that predicted by classical effects and the energy of peak density is different with 450 keV. The role of relativity is verified as an explanation.7

In this paper, the experimental anomaly and the role of relativity are studied in detail with both high-resolution Monte Carlo (MC) and particle-in-cell (PIC) simulations. To enhance significantly the resolution and accuracy, 100 times more particles and larger grid numbers are used than that in the letter-size paper.7 The thermal and beam Doppler effects on the fusion-produced alpha particle distribution are calculated. The slowing down process of the Doppler-broadened MeV alphas is then examined with a MC code that is developed for studying the classical collision effects. The results indicate the significant disagreement with the experimental measurement. Thus, an instability mechanism such as ion cyclotron instability is needed for the anomalous broadening. Of a few possible cyclotron instabilities2–7,18–26 considered, the quadratic type of relativistic ion cyclotron instability is identified. The resultant relativistic gyrobroadeened distribution obtained from PIC simulation is three times broader than that of the classical Doppler effects. While this relativistic gyrobroadeened distribution is used for the MC calculation and the time delay in diagnostic pellet releasing and flight is also included, the result is in good agreement with the experimental measurement. Both the spectral width and the energy of peak density agree. To test the goodness of fit, the reduced chi squares $\chi^2$ are calculated. Only the reduced chi square of the relativistic case can be one.

Due to a small mass deficit, the relativistic instability of alpha particles is fundamentally different with that of protons.4,6,26 The relativistic instability is in a quadratic power of the resonance. As a result, the alphas are selectively gyrobroadeened,5 which is in sharp contrast to the anomalous thermalization of protons.6 In addition, the relativistic instabilities provide an explanation26 for the experimentally observed ion cyclotron emissions. The emission is an important clue to understand the MeV ion and plasma dynamics. All of these significant consequences are caused by the relativity even when the Lorentz factor of MeV ions is extremely close to unity (e.g., 1.000 94 for the alphas.) Thus, the verification of the essential role of relativity in realistic devices for the
Doppler-broadened MeV ions to drive the instabilities and for their dynamics to be significantly affected is important because of the novel physics, interesting consequences, and potential applications.

This study has immediate application in fusion plasmas. As we just said, the fusion-produced alphas are believed to be classically slowed down by collisions. However, we suggest that the relativistic gyrobroadening may occur before the collision effect and determine the characteristics of the alpha particle source for other processes. So far, the alpha particle density is low. Since it will be high in the burning devices of next generation, the relativistic ion cyclotron instability will be stronger and the consequences such as the alpha particle and plasma transport in both velocity and density make down. The relativistic instabilities are an efficient means for waves to interact and exchange energy with the alpha particles. In fact, a recent calculation shows that the efficiency of energy extraction due to the relativistic effect compared with the nonrelativistic effect is proportional to $J_n/J'_n$, where $J_n$ is the Bessel function of first kind of order $n$ and the superscript $'$ stands for derivative. The relativity plays a dominant role because the relativistic instability peaks at $J'_n=0$. Thus, the relativistic mechanism is more effective for the energy tapping.

The beam-blip experiment is explained in the next section. Section III discusses the birth distribution of fusion-produced alpha particles including Doppler-broadening effects. Section IV addresses the Monte Carlo simulation for calculating the classical slowing down. The experimental anomaly is revealed in Sec. V. Possible ion cyclotron instabilities are discussed in Sec. VI. The relativistic gyrobroadening as an explanation for the anomaly is studied in Sec. VII. Section VIII is the summary.

II. BEAM-BLIP EXPERIMENT

The collisional slowing down time of 3.5 MeV alpha particles in the typical plasmas is a fraction of 1 s, while the time scale of relativistic gyrobroadening is much shorter. Thus, the relativistic ion cyclotron instability may be driven by the energetic alpha particles before their slowing down, so as the gyrobroadening of the alpha particles. However, when the time becomes longer, the gyrobroadened alpha particle distribution will be complicated by the collisions with thermal plasmas and the alpha particle transport. For measuring the alpha energy spectrum near their birth in order to quantify possible instability effects, the TFTR experiment employing a very short pulse of triton beam interacting with thermal deuterons was proposed.

In the beam-blip experiment (shot No. 86299) (Refs. 16 and 17) of TFTR, a 95 keV neutral triton beam that lasts for 100 ms is injected to fuse with thermal deuterons. At 20 ms after the end of the triton beam, a diagnostic boron pellet is sent for measuring the energy spectrum of the fusion-produced alpha particles near their birth. The alpha particles with pitch angle around $\nu/\nu_0=-0.048\pm 10^{-3}$ near the plasma core are detected by the pellet charge exchange diagnostic with neutral particle analyzer. The bounce points of these alpha particles are less than one alpha gyroradius away from the midplane; that is, they are deeply trapped. The energy spectrum between 1.75 and 3.5 MeV is measured. The result is given in Table I. The measured alpha particle densities and their normalization values are listed. The error is calculated from the error bar defined as the standard derivation divided by the data $\sigma/y=N^{-1/2}=0.7,0.3,0.3,0.3,0.5$ starting from the lowest-energy point, where $N$ is the number of counts.

The experimental data at 1.75 MeV are uncertain because the signal is weak.

The plasma parameters near the core are $B=4.9$ T, the thermal deuterium density $n_\beta=1.1 \times 10^{13}$ cm$^{-3}$, the electron density $n_e=3 \times 10^{13}$ cm$^{-3}$, their temperatures $T_\beta=T_e=6$ keV, and the alpha density at the end of the neutral beam injection (NBI) is $n_\alpha=7.0 \times 10^{5} n_e$. The classical slowing down frequency calculated later is 3.5 s$^{-1}$ for the beam ions and the alpha particles. The collision frequency of pitch angles scattering is 2.3 s$^{-1}$ for the beam ions and 0.016 s$^{-1}$ for the 3.5 MeV alphas. Since the time of classical slowing down is larger than those of the beam and the pellet, the data can reveal information about the early alpha energy spectrum.

III. DOPPLER-BROADENED ALPHA BIRTH DISTRIBUTION

We are going to study the birth distribution of alpha particles. During thermonuclear fusion reactions, the fusion of a deuteron and a triton produces an alpha particle with a kinetic energy of 3.5 MeV and a neutron of 14.1 MeV. The reaction can be written as

$$D + T \rightarrow a(3.5 \text{ MeV}) + n(14.1 \text{ MeV}).$$

The fusion-produced alpha particle has no preference in direction at the center-of-mass frame. If the reacting nuclei have no momentum, the distribution of the alpha particles is a cold shell in three-dimensional momentum space. When the reacting nuclei have a temperature, the alpha particle distribution is broadened by the Doppler effect. Their energy distribution becomes an isotropic warm shell with a full width at half maximum (FWHM) energy spread of

<table>
<thead>
<tr>
<th>Alpha energy (MeV)</th>
<th>Alpha density (10 MeV$^{-1}$ m$^{-3}$)</th>
<th>Alpha density (normalized)</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.75</td>
<td>0.08</td>
<td>0.096</td>
<td>±0.068</td>
</tr>
<tr>
<td>2.12</td>
<td>0.694</td>
<td>0.837</td>
<td>±0.251</td>
</tr>
<tr>
<td>2.53</td>
<td>0.829</td>
<td>1.00</td>
<td>±0.300</td>
</tr>
<tr>
<td>2.99</td>
<td>0.692</td>
<td>0.835</td>
<td>±0.250</td>
</tr>
<tr>
<td>3.5</td>
<td>0.3</td>
<td>0.362</td>
<td>±0.181</td>
</tr>
</tbody>
</table>
in the center-of-mass frame, that is, the laboratory frame, where \( E_{\alpha 0} = 3.5 \) MeV the birth alpha energy, \( T \) the plasma temperature, \( m \) the ion mass, and the subscripts \( \alpha, D, \) and \( T \) stand for alpha, deuteron, and triton, respectively. For the plasma temperature of 6 keV, the energy spread is 430 keV.

For beam tritons reacting with thermal deuterons, the center-of-mass frame is moving with the velocity,

\[
V_{\text{c.m.}} = V_b \frac{m_T}{m_D + m_T},
\]

in the laboratory frame where the subscripts \( b \) and c.m. stand for beam and the center of mass, respectively. The velocity of a triton beam with an energy of 95 keV is \( 2.46 \times 10^8 \) cm/s so that the velocity of the center of mass is \( 1.48 \times 10^8 \) cm/s. The alpha particle distribution is not symmetric in the laboratory frame. The velocity perpendicular to the beam direction \( \hat{z} \) remains with the same thermal width while its energy peak shifts to

\[
E_{\alpha} = E_{\alpha 0} + E_b \frac{m_m m_T}{(m_D + m_T)^2},
\]

where \( E_b \) is the beam energy. The shift of energy peak is only 46 keV that is much smaller than the alpha birth energy of 3.5 MeV. The parallel velocity shifts up or down its energy depending on its direction along or against the beam; that is, the energies of the distribution peak become

\[
E_{\alpha} = E_{\alpha 0} \left( 1 \pm 2 \frac{\sqrt{m_d m_T}}{m_D + m_T} \frac{E_b}{E_{\alpha 0}} \right),
\]

in along (+) and against (-) beam directions. Thus, the corresponding width of the dual energy peaks in parallel direction is

\[
\Delta E_{\alpha} = 4 \sqrt{E_{\alpha 0} E_b} \frac{\sqrt{m_d m_T}}{m_D + m_T}.
\]

The difference of the dual energy peaks in parallel direction is 1.6 MeV that is not small.

Figure 1 shows the scatter plot of the birth alpha particles in momentum space including only the Doppler effects. Because more than \( 5 \times 10^8 \) particles are used in the simulation, only some of them, 1 in every 23, are plotted. The alpha particles have an energy spread of 430 keV, and a Doppler-shifted axial momentum with the corresponding difference of the dual energy peaks in parallel direction being 1.6 MeV as predicted by the analysis.

### IV. MONTE CARLO SIMULATION

There is a period of time between the birth of alpha particles and the diagnostics. During this time period, alpha particles collide with thermal plasmas so that they lose their kinetic energy and change their moving direction, so as the pitch angle between their velocity and confining magnetic field. A MC code is developed for studying both classical slowing down and pitch angle scattering. Since the time period is short, the alpha particle transport in real space is not included.

The classical slowing down\(^{38} \) of the alpha particle energy is dominated by the collision with thermal electrons and is determined by

\[
\frac{d}{dt} E_{\alpha} = - \nu_c E_{\alpha}, \quad (7)
\]

\[
\nu_c = 3.2 \times 10^{8}\frac{\lambda_{ae} T_e}{m_e}, \quad (8)
\]

\[
\lambda_{ae} = 24 - \ln \left( \frac{\sqrt{n_e}}{T_e} \right), \quad (9)
\]

where \( \nu_c \) the classical slowing down frequency, \( \lambda_{ae} \) the Coulomb logarithm for collision between the alpha particles and the electrons, \( n_e \) the electron density in unit of cm\(^{-3} \), and the electron temperature in unit of eV. The formulas given above are valid for the kinetic energy of alpha particle less than a few MeV and much larger than the electron thermal energy of a few keV temperature; that is, the kinetic energy of alpha particle is much larger than that of electron but the alpha velocity is much slower than the electron velocity. For the electron density of \( 3 \times 10^{13} \) cm\(^{-3} \) and the electron temperature of 6 keV as in the experimental plasma parameters, the Coulomb logarithm is 17 and the classical slowing down frequency is 3.5 s\(^{-1} \). Thus, the diagnostic time is much shorter than the classical slowing down time so that the birth alpha particle energy spectrum may be revealed by the experimental measurement.

The pitch angle between the velocity of alpha particle and the confining magnetic field is defined as \( \xi = \nu_s / \nu \). After collision, the alpha particle velocity may change so as its pitch angle. During the Monte Carlo simulation, the pitch angle of alpha particle is updated in each time step \( \tau \). Each pitch angle change is small by using a small time step. The change of the pitch angle is determined by\(^{38,39} \)

\[
\Delta \xi = - \nu_c \tau \xi + \sigma \sqrt{\nu_s} (1 - \xi^2 + \nu_c \tau), \quad (10)
\]

\[
\nu_c = \nu_c^{\alpha D} + \nu_c^{\alpha e}, \quad (11)
\]
\[ \nu_{1/a}^{DD} = 3.6 \times 10^{-7} \frac{\lambda_{ad} n_D}{E_{1/a}^{3/2}} , \]  
\[ \nu_{1/e}^{ae} = 3.2 \times 10^{-9} \frac{\lambda_{ae} n_e}{\sqrt{T_e E_{1/e}}}, \]  
\[ \lambda_{aD} = 13 - \ln \left( \frac{\sqrt{n_e}}{T_e E_{1/e}} \right), \]

where \( \sigma \) is a random number with the same probability of either \(-1\) or \(1\) (Ref. 39) instead of in between, \( \nu_{1/a} \) is the sum of the alpha collisions with the thermal deuterons and the electrons, and \( n_D \) is the deuteron density. Both the alpha particle energy and the electron temperature are in the unit of eV, and the density is in the unit of \( \text{cm}^{-3} \). It is again considered that the kinetic energy of alpha particle is much larger than that of electron but the alpha velocity is much slower than the electron velocity. For the typical experimental plasma parameters, the Coulomb logarithm of the alpha particles colliding with the thermal deuterons is 17. The collision frequency of alpha and deuteron is 0.01 s\(^{-1}\) and that of alpha and electron is 0.006 s\(^{-1}\) so that the overall collision frequency for pitch angle scattering is 0.016 s\(^{-1}\). Thus, the pitch angle scattering is not important because the time period between the alpha particle birth and the diagnostic is much shorter than the collision time. However, it is included in the MC code.

In the MC simulations, the alpha particles are loaded for 100 ms with its birth rate linearly increased with time as indicated from the neutron production rate measured in the experiment.\(^{16,17}\) To benchmark the MC code, a MC simulation employing an overbroadened Fokker-Planck post-TRANSP (FPPT) alpha source with the temperature of 44 keV (Refs. 27 and 40) is calculated. This overbroadened alpha source distribution corresponding to an energy spread of 1.2 MeV is used by previous FPPT calculations\(^{16,17,27}\) for all the long-beam and beam-blip experiments.\(^{16,17}\) The FPPT calculations using a time-dependent source obtained from a transport code\(^{28}\) (TRANSP) includes the toroidal and related nonideal effects. The slowed-down spectrum with their pitch angle between −0.1 and 0.1 obtained from the MC simulation and the FPPT result are shown on Fig. 2. The good agreement between the MC and FPPT simulations validates the MC code. This also indicates that the toroidal and nonideal effects included in FPPT may not be important for the energy spectrum of the measured deeply trapped alpha particles.

V. EXPERIMENTAL ANOMALY

The classical slowing down energy spectrum of the alpha particles will be compared with the experimental measurement. The Doppler-broadened alpha particles (some of them, 1 in every 23, are shown on Fig. 1) are used as the source for the MC simulation. The slowed-down alpha particles with their pitch angle between −0.049 and −0.047 are measured at the time of 120 ms. Their energy spectrum and the experimental measurement are shown on Fig. 3. The width of the classical spectrum is too narrow by 1 MeV as compared with the experimental spectrum, while its peak energy is 450 keV higher.

In the FPPT simulations,\(^{16,17,27}\) the Doppler-broadened alpha particle source with the plasma temperature as high as 44 keV is used to produce much broader energy spectrum. Because the time scale of the beam blip is much shorter than the collision times of slowing down and pitch angle scattering as well as only the alpha particles at perpendicular direction are measured, the perpendicular alpha energy spread has been significantly overestimated. Thus, it is not valid to compare the FPPT result to the measurement for the beam-blip experiment.

The disagreement indicates that the alpha particles may not be classically slowed down. Plasma instability may be needed to provide the mechanism responsible for the anomalous broadening of the alpha particle energy spectrum. Because ion cyclotron emissions excited by fast ions have also been observed in tokamaks,\(^{18,19}\) it is natural to consider ion...
cyclootron instability as the possible mechanism to cause such large broadening of alpha particle energy spectrum within a very short period of time.

VI. POSSIBLE CYCLOTRON INSTABILITIES

Two classical and one relativistic mechanisms have been proposed for explaining the experimentally observed ion cyclotron emissions. The magnetoacoustic instability is due to the coupling of electrostatic and electromagnetic modes and the Landau growth with the additional magnetic gradient drift. This requires the resonance of particular wavelength and Larmor radius. Thus, the unstable wave spectrum becomes very narrow. Also, the instability is weak as compared with that of the relativistic effect for the magnetoacoustic wave.

The other classical mechanism proposed for the alpha particles to drive electromagnetic ion cyclotron wave is an inverse-Landau-type instability. This is also a narrow effect on changing the energy spectrum. The growth rate is proportional to the density of the alpha particles. Since the alpha particle density is low, the growth rate is far too weak. Thus, these two classical mechanisms cannot produce the large energy broadening at MeV level, especially with low wave amplitude.

The relativistic electrostatic ion cyclotron instabilities also provides an explanation for the experimentally measured ion cyclotron emissions. The relative amplitudes of different harmonics agree well with the experimental measurements in Joint European Tokamak. The amplitude of the first fast ion harmonic scaled with the fast ion density is 0.9±0.1 over six order of magnitude, while the prediction of particle-in-cell simulation is 0.86. The recent study of relativistic electromagnetic ion cyclotron instabilities discovers the Alfvénic behavior, as seen in the ion cyclotron emission measurements in the TFTR experiments.

The mechanism of the relativistic ion cyclotron instabilities driven by fusion-produced fast ions is the gyrophase bunching induced by the resonant interaction between the wave and the fast ion harmonic cyclotron motion that includes the cyclotron frequency dependence on the relativistic mass variation effect. For the slow ion harmonic cyclotron motion to be involved in the resonance for driving the instabilities, the harmonic cyclotron frequency of the fast ion is required to be smaller than that of the slow ion; that is, the frequency mismatch has to be negative. But, due to mass deficit, the frequency mismatch between the 3.5 MeV fast alpha particles and the thermal deuterons is positive. Thus, the relativistic ion cyclotron instability can only be driven at high harmonics, where the slow ions can be treated as a cold plasma so that their high harmonics are not involved in the resonant interaction.

VII. RELATIVISTIC GYROBROADENING

The relativistic ion cyclotron instability can be described by the dispersion relation, which is derived by relativistic kinetic theory and can be written as

\[ A = 1 - \frac{\omega_{pD}^2 S_1}{n^2 \Omega_{c,a}^2 - \Omega_{c,D}^2} \]  

\[ B = \frac{\omega_{pD}^2}{2 \omega_{c,a}} (J_{n-1}^2 - J_{n+1}^2) \]  

\[ C = \frac{\omega_{pD}^2 n^2 \omega_{c,a}^2}{k^2 c^2} (J_n^2) \]  

where \( S_1 = 2 I_1(\lambda_D)(e^{-\gamma_{D}^2/\lambda_D}) \), \( \omega_p \) is the plasma frequency, \( I_1(\lambda_D) \) is the modified Bessel function of the first kind of order 1, \( \lambda_D = k^2 T/m_D \Omega_{c,D}^2 \), and \( T \) is the slow ion temperature,

\[ \langle J_n^2 \rangle = \frac{2 \pi P_1}{\omega_{c,a}} P \int dP \frac{J_n^2(k \rho_a)}{N} \]  

The birth kinetic energy of the alpha particles has been treated as a constant in the theoretical analysis for simplicity, because the alpha particle distribution with the Doppler broadening effect is not far away from that without. The modes with \( k_1 = 0 \) are considered here because they are most unstable as driven by the relativistic reactive-type instability. Finite \( k_1 \) can cause Landau growth. But, the growth rate is proportional to the alpha density and thus weak and it is a local effect on shaping the alpha energy spectrum so that it is neglected here.

The instability is driven by the second-order resonance between the wave frequency and the harmonic cyclotron frequency of the alpha particles. The instability growth rate is given by the unstable solution of this quadratic equation. The real part of the wave frequency is very close to the harmonics of alpha cyclotron frequency. The contribution from the first harmonic of the thermal deuterons determines the sign of cold plasma dielectric \( A \) and, thus, the stability. The first unstable condition \( A > 0 \) determines the harmonic number threshold for instability,

\[ n > \frac{\sqrt{\omega_{pD}^2 S_1 + \omega_{c,D}^2}}{\omega_{c,a}} \]  

This condition indicates that the wave frequencies of the quadratic instability are at the lower hybrid regime; that is,

\[ \omega \sim n \omega_{c,a} > \sqrt{\omega_{pD}^2 S_1 + \omega_{c,D}^2} \]  

The other unstable condition is \( B^2 - 4 AC < 0 \) or

\[ A \left( \frac{16 n^2 \omega_{c,a} \omega_{pD}^2}{k^2 \Omega_{c,a}^2} \right) \left( \frac{J_n^2}{J_{n-1}^2 - J_{n+1}^2} \right)^2 > \frac{\omega_{pD}^2}{\omega_{c,a}^2} \]  

This implies that higher alpha density can stabilize the harmonic ion cyclotron waves at certain wavelengths. When \( B^2 \leq 4 AC \), the peak growth rate for each unstable harmonic \( n \) is

\[ \omega_{\text{max}} \sim \frac{n \sqrt{J_n^2}}{k c} \sqrt{A} \]  

and the corresponding real part of the wave frequency is
The maximum growth rate is scaled as the square root of the alpha particle density.

A particle-in-cell (PIC) code, which is one dimension in space and three dimension in momentum, is developed to simulate possible instability driven by the Doppler-broadened alpha particles as well as its consequences. Both the wave vector and the wave electric field are in the $\hat{x}$ direction. The system is periodic with a length of 4098 cells. Each cell size $\Delta x$ is 0.0225 cm. Only the wave modes from 1 to 100 are kept in order to reduce the numerical noise. The unit time is $t_0=\Omega_{CD}^{-1}$, and the time step is 0.025. The number of simulation particles for deuterons (alphas) is 2,598,112 (5,038,848). In the PIC simulation, the initial distribution of the alpha particles is given as the Doppler-broadened distribution shown in Fig. 1, where only some alpha particles (1 in every 23) are plotted. Because 23 is a prime number different with other prime numbers used in the quiet start loading of the alpha particles, this figure can represent the distribution of all the alpha particles, without loss of generality.

Figure 4 shows the history of total field energy. The total field energy grows from the noise to about $10^{-5}$ of the initial kinetic energy of alpha particles. When we turn off the relativistic effect by using Newton equation instead of Lorentz equation for particle motion, the total field energy remains at the noise level. This verifies that the wave growth is caused by the relativistic effect. To further clarify the relativistic effect, the history of the field energy of the most unstable mode 44 is shown in Fig. 5. The PIC simulation result indicates that the mode 44 with $k_{p\omega}=16.5$ grows with a rate of 0.46% $\Omega_{CD}$, that is slightly smaller than 0.50% $\Omega_{CD}$ of another PIC simulation beginning with a cold shell distribution while the relativistic kinetic theory predicts 0.46% $\Omega_{CD}$. The good agreement verifies that the relativistic instability is the quadratic type of relativistic ion cyclotron instability discussed earlier.

The resultant scatter plot of the alpha particles in momentum space obtained at the end of the simulation is shown on Fig. 6. Again, some representative alpha particles of 1 in every 23 are plotted. There is a threshold for the alpha particles to be involved in driving the instability and efficient wave particle interaction; that is, the interaction is selective. Only the alpha particles with perpendicular momentum larger than 140$m_\alpha\Delta\omega_{CD}$ are involved in the interaction. For those selected, some of their perpendicular momenta are slowed down to 140$m_\alpha\Delta\omega_{CD}$, while some are accelerated up to 320$m_\alpha\Delta\omega_{CD}$. When the relativistic effect is turned off, the momentum distribution of the alpha particles remains as that at the initial (not shown). Thus, the further selective gyro-broadening of the Doppler-broadened alpha particles is due to the relativistic effect. Newly born alpha particles added after the wave saturation experience a similar gyro-broadening process and result within a much shorter time.

The energy spectra of both the relativistic gyrobroadened alphas and the Doppler-broadened alphas with their pitch angle between $-0.049$ and $-0.047$ are shown on Fig. 7 for comparison. The width of the alpha particle energy spectrum resulted from the relativistic instability is three times that of the Doppler broadening, in addition to the lower energy of the peak density. The relativistic gyrobroadening effect

\[
\frac{\omega_r}{\omega_{ca}} \sim n - \frac{B}{2A\omega_{ca}}.
\]
dominates in determining the alpha particle distribution. To evaluate the sensitivity of the results for different plasma parameters, the PIC simulations with different alpha particle densities or beginning with a cold shell distribution have been done. Although the growth rates of the unstable waves are different, the gyrobroadened distribution remains about the same.

The relativistic gyrobroadened alpha particles are then used as the source for the MC simulation. Because the collision time of the pitch angle scattering is about 500 times longer than the experiment and thus simulation time period, the change of pitch angle is very small. Without loss of generality, only alpha particles with their pitch angle between $-0.15$ and $0.05$ are loaded as the input sample for the MC simulation. Different pitch angle ranges are also used for the input sample and the results remain almost the same. The alpha particles chosen from the input sample are loaded for 100 ms with their density linearly increasing with time as in the experiment.\textsuperscript{16,17} The number of alpha particles loaded in the MC simulation is $1/10^8$. The slowed-down spectra of both the gyrobroadened alphas and the Doppler-broadened-only alphas with their pitch angle between $-0.049$ and $-0.047$, and the experimental measured data are shown on Fig. 8. As we discussed earlier, the experimental data at 1.75 MeV are uncertain due to weak signal.\textsuperscript{35} The peak energy of the slowed-down Doppler-broadened spectrum is 450 keV higher than the experiment, while its width is too narrow by 1 MeV. Both the width and the peak of the slowed-down relativistic gyrobroadened spectrum fit the experimental data better. However, a systematical difference on the density peak and the profile exists.

As we found, before the charge-exchange measurement, the releasing of the diagnostic pellet requires 14–18 ms of time and then it takes 2–4 ms for the pellet to fly from the edge to the core of the tokamak.\textsuperscript{35} Figure 9 shows the calculated slowed-down alpha particle spectra including the 20 ms time delay, the experimental data, and the $\chi^2$-optimized gyrobroadening curve with the theoretical error bars discussed later. The spectrum based on Doppler-broadening mechanism is still too narrow while the energy of its density peak is 250 keV higher than that of the experiment. As for the gyrobroadening mechanism, the data at 1.75 MeV are higher than that of the experiment. There are a few possibilities. These experimental data are uncertain because of weak signal.\textsuperscript{35} The spatial and detail time dependence of the alpha productions, the alpha transport in real space, and the nonuniformity of magnetic field\textsuperscript{34} are not considered in the simulations. Other than that, both the spectral width and the energy of peak density of the slowed-down relativistic spectrum remarkably agree with the experimental measurement.

The reduced chi square $\chi^2$ (Ref. 42) is a test for goodness of fit. A curve being able to produce $\chi^2=1$ is a good fitting. Since both the calculated and measured data are unnormalized, the calculated curve can be multiplied by an arbitrary constant. The best value of this constant $a$ is determined by the optimization of $\chi^2$. The expectation value of $\chi^2$ can at best be one. Assuming the same statistics, the theoretical value $\sigma/y$ equals that of the experiment. As shown in
Table II, the optimized $\chi^2_{\text{rel}}$ for the gyrobroadening and the Doppler broadening are 1.0 and 8.5 with $\alpha$ being 0.93 and 630, respectively. For reference, the optimized $\chi^2_{\text{rel}}$ for the FPPT with an overestimated ion temperature is 2.2 with $\alpha=2.4$. The agreement between the relativistic gyrobroadening result and the experimental measurement is of statistical significance. Since only the relativistic gyrobroadening result can yield $\chi^2_{\text{rel}}=1$, the relativistic mechanism provides the sole explanation of the experimental anomaly.

### VIII. SUMMARY

The experimental anomaly of the measured much broader alpha energy spectrum has been studied in detail by high-resolution Monte Carlo and particle-in-cell simulations. It is concluded that the relativity may have played an important role for the alpha particles to drive the relativistic ion cyclotron instability and hence to be selectively gyrobroadened in the TFTR experiment, although their nonsymmetric Doppler-broadened distribution has a large width of dual axial energy peaks and their Lorentz factor is extremely close to unity. The width of the resultant alpha particle energy spectrum in the perpendicular direction is three times that of the Doppler broadening. Since both the spectral width and the energy of peak density agree as well as the reduced chi square can be one, the MC/PIC result is in good agreement with the TFTR measurement,\textsuperscript{16,17} that may also provide the first possible experimental evidence of the relativistic gyrobroadening effect, in which the critical role of relativity is confirmed again by turning off the relativity.

This study has immediate applications in fusion plasmas, in addition to the importance in fundamental plasma physics. The relativistic instability and resultant gyrobroadening may determine the alpha particle source distribution for classical slowing down and other processes.\textsuperscript{12–15} The possibility to tap the alpha energy through the relativistic ion cyclotron instability is encouraged. So far, the alpha particle density is low. Since it will be high in the burning devices of next generation, the relativistic instability will be stronger and the consequences such as the alpha particle and plasma transport in both velocity and real spaces remain to be open issues of great importance. More beam-blip experiments for measuring the new-born alphas much earlier than their slowing down time are called for. In addition to fusion, the relativistic effects may also have important applications in space plasma studies,\textsuperscript{31} e.g., as an energy broadening or thermalization mechanism in solar corona.\textsuperscript{32}

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35. S. S. Medley (private communication, 2001).
36. The alpha density at 10 ms after the beginning of the NBI is $n_\alpha=7.0 \times 10^{22} n_e$, that is used for the PIC simulation, and $n_\alpha E_x=0.1 n_\alpha T_D$. The relativistic gyrobroadening results of the PIC simulations for different alpha densities remain about the same.

\[ \text{TABLE II. The optimized reduced chi squares } \chi^2 \text{ and the optimization constants } a. \]

<table>
<thead>
<tr>
<th></th>
<th>Relativity</th>
<th>Doppler only</th>
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<tbody>
<tr>
<td>$\chi^2$</td>
<td>1.0</td>
<td>8.5</td>
</tr>
<tr>
<td>$a$</td>
<td>0.93</td>
<td>630</td>
</tr>
</tbody>
</table>
40 N. N. Gorelenkov (private communication, 2001).
41 K. R. Chen, Bull. Am. Phys. Soc. 45, 132 (2000); 46, 190 (2001). The PIC simulations show that the relativistic instability survives and the relativistic gyrobroadening remains the same in both sinusoidal and 1/r non-uniformities of the magnetic field.