Effects of a guide field on the evolution of a current sheet

C. L. Tsai  
Institute of Space Science, National Central University, Jhongli, 320 Taiwan; and Earth Dynamic System Research Center and Department of Physics, National Cheng Kung University, Tainan, 701 Taiwan

L. C. Lee  
Institute of Space Science, National Central University, Jhongli, 320 Taiwan

B. H. Wu  
National Space Organization, Hsinchu, 300 Taiwan

(Received 7 August 2006; accepted 8 September 2006; published online 16 October 2006)

The evolution of an initial current sheet is studied by using the set of one-dimensional (1D) magnetohydrodynamic equations. In a simulation of the 1D Riemann problem along the z direction, the initial magnetic field is chosen as $B(z) = -B_{0z} \tanh(z/\delta)\hat{z} + B_x \hat{x} + B_{0z} \hat{z}$, where $B_{0z}$ denotes the antiparallel component and $B_x$ is called the guide field of current sheet. For the $B_x = 0$ case, a pair of slow shocks is formed and propagates away from the current sheet. For $B_x \neq 0$ (even for a very small $B_x$), it is found that a pair of slow shocks and an additional pair of time-dependent intermediate shocks (TDISs) are formed. TDISs are not present in the $B_x = 0$ case, indicating that the case with $B_x = 0$ is a singular case. In this paper, an attempt is made to reconcile the $B_x = 0$ case with small nonzero $B_x$ cases by examining the early time evolution of TDISs and slow shocks. The early current evolutions for the $B_x = 0$ case and for small $B_x$ cases are found to be very similar. For $B_x \neq 0$ cases, the plasma density and pressure are found to increase and the magnetic field decreases across TDISs. The dependence of current sheet evolution on initial $B_x$ (or $\phi_{\infty}$) and the plasma beta $\beta_{\infty}$ and also examined, where $\phi_{\infty} = \tan^{-1}(B_x/B_{0z})$ is the initial angle between the tangential magnetic field and the x axis, and the subscript $\infty$ denotes the location far from the current sheet. The rotation angle ($\Delta \phi$) of tangential magnetic field across TDIS develops gradually with time and reaches the final value $\Delta \phi_{\text{final}} = 90^\circ - \phi_{\infty}$. For small $B_x$ ($\phi_{\infty} \to 0$), it is found that the time reaching the final state is very long. Both pressure and temperature downstream of the slow shock decrease with $\phi_{\infty}$ and increase with plasma beta $\beta_{\infty}$. With a decreasing $\phi_{\infty}$ or an increasing $\beta_{\infty}$, the Alfvén Mach number ($M_A = V_{\infty}/V_A$) associated with slow shocks increases, and the amount of magnetic energy converted into kinetic energy also increases. © 2006 American Institute of Physics.

[DOI: 10.1063/1.2358496]

I. INTRODUCTION

Magnetic reconnections frequently take place at the current sheet that separates two plasma regions that have an antiparallel magnetic field component.\textsuperscript{1–7} The inflow pushes the antiparallel magnetic field lines toward the center of current sheet. Once the magnetic field lines are broken and reconnected, the incoming magnetic energy is released, leading to formation of high-speed outflow regions. Magnetic reconnection plays an important role in solar flares, corona heating, geomagnetic storms, and tokamaks.\textsuperscript{1–20} It provides a fast and effective mechanism for conversion of magnetic energy to kinetic energy in space and laboratory plasmas.

Various magnetohydrodynamic (MHD) discontinuities are generated in magnetic reconnection.\textsuperscript{3,4,21–25} A layered structure that contains several discontinuities and waves is formed in the high-speed outflow region.\textsuperscript{3,4,23–25} Consider the case in which the initial current sheet is located at $z = 0$. In the coplanar case ($B_x = 0$) with equal plasma density, magnetic field strength, and antiparallel magnetic field ($B_x$) on the two sides of initial current sheet, magnetic reconnection is triggered in a small diffusion region where two slow shocks (SS) are generated.\textsuperscript{3,5,6,25,27} In the reconnection layer, which consists of two pairs of slow shocks and two plasma outflow regions, the magnetic energy is converted into the bulk flow and thermal energy. For the noncoplanar case with a nonzero guide field ($B_x$), the reconnection layer can have rotational discontinuities (RDs), contact discontinuities, slow shocks, and slow expansion waves in the ideal MHD formulation.\textsuperscript{25} In the ideal MHD case, the plasma density, pressure, and magnetic field strength across a rotational discontinuity (RD) are conserved, but the direction of tangential magnetic field can be rotated.\textsuperscript{26} The rotational discontinuity (RD) is a nonlinear intermediate-mode structure through which the normal component of plasma flow velocity relative to the discontinuity is equal to the normal component of Alfvén velocity.

However, the steady-state rotational discontinuities cannot exit in the presence of a finite resistivity. In the dissipative MHD, rotational discontinuities are replaced by time-dependent intermediate shocks (TDISs), which belong to a new class of time-dependent, localized shock-like structure.\textsuperscript{28–30} TDISs do not obey Rankine-Hugoniot conditions since they violate coplanarity. Across a TDIS, the downstream plasma temperature, pressure, and density increase, and the downstream magnetic intensity decreases.
The direction of tangential magnetic field rotates slightly. The propagation speed of TDIS is also equal to the normal component of Alfvén velocity. However, the width of TDIS expands self-similarly as $t^{1/2}$ and the strength (density and field jumps) of TDIS decreases with time. The TDIS will gradually evolve to a RD, as $t \to \infty$, with infinite width and equal field strength on the two sides. The TDIS can be considered as a time-evolving finite-amplitude Alfvén wave. Wu and Kennel\textsuperscript{32} derived their structural relations, similar to the Rankine-Hugoniot relations, between the plasma properties and magnetic field, which can help to identify TDIS at the magnetopause, in the solar wind, and elsewhere in space. The steady intermediate shocks and TDISs play the role of rotational discontinuities in reconnection layer. The formation and evolution of MHD discontinuities in two-dimensional (2D) magnetic reconnection can be studied by the one-dimensional (1D) Riemann problem.\textsuperscript{21,23–25,27,30,32–36} For the 2D reconnection configuration in the $x$-$z$ plane, waves, and discontinuities in outflow region are reduced to $\pm z$ propagations in the 1D Riemann problem, as shown in Fig. 1, in which both $B_z=0$ case and $B_z \neq 0$ case are plotted.

For the $B_z=0$ case, the top panel in Fig. 1(a) is an illustration for the formation of slow shocks in the outflow region of magnetic reconnection in the upper $x$-$z$ plane. The magnetic field line reconnected at $t=0$ is convected to $x_1$ and $x_2$ at time $t_1$ and $t_2$, respectively. A pair of slow shocks, $XS$ and $XS'$, emanates from the point $X$. The middle panel shows the time evolution of magnetic field $B_x$ profile in the 1D initial value problem which corresponds to the 2D configuration of reconnection layer. The slow shocks at $t=t_1$ ($t=t_2$) are located at $S_1$ and $S_1'$ ($S_2$ and $S_2'$). In case (b) with $B_z \neq 0$, the top panel is an illustration for the formation of TDISs and slow shocks in the outflow region of magnetic reconnection. The magnetic field line reconnected at $t=0$ is convected to $x_1$ and $x_2$ at time $t_1$ and $t_2$, respectively. A pair of TDISs, $XI$ and $XI'$, and a pair of slow shocks, $XS$ and $XS'$, emanate from the point $X$. The middle panel and bottom panel show time evolutions of magnetic field $B_x$ and $B_y$ profiles in the 1D initial value problem. The TDISs at $t=t_1$ ($t=t_2$) are located at $I_1$ and $I_1'$ ($I_2$ and $I_2'$). The $B_y$ profile (bottom panel) shows that slow shocks at $t=t_1$ ($t=t_2$) are located at $S_1$ and $S_1'$ ($S_2$ and $S_2'$). (c) A 3D plot of reconnected magnetic field lines for the $B_z \neq 0$ case, showing the presence of TDISs ($I$ and $I'$) and slow shocks ($S$ and $S'$).
formed at the reconnection point $X$ at $t=0$. The magnetic field line reconnected at $t=0$ is convected to $x_1$ and $x_2$ at times $t_1$ and $t_2$, respectively. A pair of slow shocks, $XS$ and $XS'$, emanates from the point $X$. The middle panel illustrates the time evolution of magnetic field profile ($B_i$) in the 1D initial value problem that corresponds to the 2D configuration of reconnection layer. For $t>0$, a pair of slow shocks propagates in the $\pm z$ direction. At $t=t_1$ ($t=t_2$), slow shocks arrive at $S_1$ and $S_1'$ ($S_2$ and $S_2'$), as can be seen from the $B_i$ profiles. Note that slow shocks $S_1$ and $S_1'$ ($S_2$ and $S_2'$) at $t=t_1$ ($t=t_2$) in the middle panel correspond to slow shocks at $x=x_1$ ($x=x_2$) in the top panel. The bottom panel shows that the time evolution of magnetic field $B_i$ profile always keeps a constant zero value. The time evolution of magnetic field $B_i$ in $B_x=0$ case is very different from the $B_x \neq 0$ case shown later.

For the $B_x \neq 0$ case, the top panel in Fig. 1(b) shows formation of TDISs and slow shocks in the outflow region of magnetic reconnection in the upper $x-z$ plane. The magnetic field line reconnected at $t=0$ is convected to $x_1$ and $x_2$ at time $t_1$ and $t_2$, respectively. A pair of TDISs, $X'I'$ and $X'I''$, and a pair of slow shocks, $XS$ and $XS'$, emanate from the point $X$. The middle panel illustrates the time evolution of magnetic field profile ($B_i$) in the 1D initial value problem. In the symmetric case with equal plasma density, magnetic field strength, and antiparallel magnetic fields on the two sides of initial current sheet, the nonzero $B_z$ component leads to a pair of slow shocks and a pair of TDISs. For $t>0$, a pair of TDISs and a pair of slow shocks propagate in the $\pm z$ direction. At $t=t_1$ ($t=t_2$), TDISs arrive at $I_1$ and $I_1'$ ($I_2$ and $I_2'$) as can be seen from the $B_i$ profiles. At $t=t_1$ ($t=t_2$), slow shocks arrive at $S_1$ and $S_1'$ ($S_2$ and $S_2'$). The bottom panel shows the presence of TDISs in the $B_x$ profile, due to the rotation of magnetic field, and the following slow shocks. The slow shocks at $t=t_1$ ($t=t_2$) are located at $S_1$ and $S_1'$ ($S_2$ and $S_2'$). The magnetic fields associated with slow shocks lie in the $y-z$ plane. Figure 1(c) shows a three-dimensional (3D) perspective view of reconnected magnetic field lines for the $B_x \neq 0$ case. The TDISs ($I$ and $I'$) rotate the direction of tangential magnetic field and switches off the upstream $B_z$ component. The coplanar slow shocks ($S$ and $S'$) have the upstream and downstream magnetic fields lying in the $y-z$ plane. The $B_z$ component decreases across the slow shocks and keeps a nonzero constant downstream of slow shocks.

In our simulation of 1D Riemann problem along the $z$ direction, the initial magnetic field is chosen as $B(z)=-B_{z,0} \tanh(z/\delta) \hat{x} + B_y \hat{x} + B_z \hat{z}$, where $B_{z,0}$ denotes the antiparallel component. The coplanar ($B_x=0$) and noncoplanar cases ($B_x \neq 0$) have been studied extensively by Lin and Lee.\textsuperscript{25} They also examined the development and evolution of intermediate shocks and TDISs in the 2D reconnection layer.\textsuperscript{24}

In the present study, we will examine in details the role of guide field ($B_x$) in the evolution of an initial current sheet. We find that any small initial $B_x$ will trigger generation of TDISs in addition to slow shocks, while $B_x=0$ is a singular case without TDISs. We attempt to reconcile the $B_x=0$ case and cases with a small initial $B_x$ by examining the early time evolution of TDISs and slow shocks. We find the similarities between the singular $B_x=0$ case and cases with small $B_x$ in early times. For $B_x \neq 0$ cases, TDISs switch off the $B_x$ component and generate a $B_z$ component as time goes on. TDISs also generate a high-speed flow in $V_y$ and $V_z$. The following slow shocks then reduce the magnitude of $B_z$ and $V_y$. Due to sharing of initial magnetic energy by the additional TDIS pair, slow shocks are found to be weaker than in those cases without TDISs. We also examine how the Alfvén Mach number ($M_A=V_{ni}/V_A$) associated with slow shock changes as a function of $B_i$ and plasma beta.

II. SIMULATION MODEL AND TWO TYPICAL CASES

We use resistive MHD equations to study the evolution of current sheet. The set of resistive MHD equations, consisting of continuity equation, momentum equation, Faraday equation, energy equation, and Ohm’s law, are given by

$$\frac{\partial \rho}{\partial t} = - \nabla \cdot (\rho \mathbf{V}), \quad (1)$$

$$\frac{\partial (\rho \mathbf{V})}{\partial t} = - \nabla \cdot \left[ \rho \mathbf{VV} + \left( \frac{B^2}{8 \pi} \right) \mathbf{I} - \frac{1}{4 \pi} \mathbf{BB} \right], \quad (2)$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \mathbf{E}, \quad (3)$$

$$\frac{\partial}{\partial t} \left( \frac{\rho V^2}{2} + \frac{P}{\gamma - 1} + \frac{B^2}{8 \pi} \right) = - \nabla \cdot \left( \frac{\rho V^2}{2} + \frac{\gamma P}{\gamma - 1} \right) \mathbf{V} - \frac{1}{4 \pi} \mathbf{E} \times \mathbf{B}, \quad (4)$$

$$\mathbf{E} = - \mathbf{V} \times \mathbf{B} + \eta \mathbf{J}, \quad (5)$$

where $\rho$, $P$, $V$, $B$, $E$, $J$, and $\eta$ are, respectively, the mass density, pressure, flow velocity, magnetic field, electric field, current density, and electrical resistivity, and $\mathbf{I}$ is a unit tensor.

Here, the specific heat ratio $\gamma$ is set to 5/3.

In our simulation, the length, magnetic field, and mass density are normalized by the background values $L_0$, $B_0$, and $\rho_0$, respectively. The pressure, temperature, and time are then normalized by $P_0=B_0^2/4\pi$, $T_0=m_j P_0/k_b \rho_0$, and $t_0=L_0/V_0$, where $V_0=B_0/\sqrt{4\pi \rho_0}$ is the background Alfvén speed, $m_j$ is the ion mass and $k_b$ is Boltzmann’s constant. The dimensionless resistivity is defined as $\eta=L_0 V_0/4\pi \sigma$, the inverse of magnetic Reynolds number $R_m^{-1}$.

In our 1D simulations, shocks propagate in the $\pm z$ direction, as shown in Fig. 1(b), and the normal magnetic field $B_z$ remains constant. The initial profile of magnetic field is set to $B(z)=-B_{z,0} \tanh(z/\delta) \hat{x} + B_y \hat{x} + B_z \hat{z}$, where $B_{z,0}=B_{z,0} \sin \theta \cos \phi_{z,0}$, $B_y=B_{z,0} \sin \theta \sin \phi_{z,0}$, and $B_z=B_{z,0} \cos \theta$. We define $\theta$ as the normal angle between the magnetic field and the $z$ axis and $\phi$ as the angle between the tangential magnetic field and the $x$ axis; i.e., $\tan \phi=B_y/B_z$. The subscript $\infty$ denotes the magnetic field far from current sheet and $\delta$ denotes the width of initial current sheet. The noncoplanar configuration is characterized by a nonzero $B_x$ or $\phi_x$. In our simulation, there are 8000 grid points with uniform grid-spacing of $\Delta z=0.5$ and the current sheet half-width is set to $\delta=2.5$. 


The resistivity is set to a constant value $\eta=2 \times 10^{-5}$. The initial pressure profile is determined from the pressure balance equation $P(z) + B^2(z)/(8\pi) = P_0 + B_0^2/(8\pi)$, where $P_0$ is related to $B_0$ by $\beta_0 = P_0/(B_0^2/8\pi)$. The initial temperature is assumed to be constant; i.e., $T(z) = T_0$. The density profile can be written as $\rho(z) = P(z)/T_0 = \rho_0 P(z)/P_0$. Therefore, the initial state of 1D Riemann problem in this study is characterized by three parameters: $\beta_0$, $\theta$, and $\phi_0$. The evolution of current sheet has also been used to study the structure of slow shocks and intermediate shocks in the presence of heat conduction.35,36

We first present a case with $B_y=0$ to be compared with a $B_y \neq 0$ case. We then pay attention to the role of $B_y$ component on evolutions of TDISs and slow shocks after reconnection onset. The final rotational angle of tangential magnetic field across TDIS is found to be related to the initial $B_y$ component or $\phi_0$ angle. The intensity of generated slow shocks and associated energy conversion across shocks are analyzed. The evolution of current sheet with different values of $\phi_0$ and $\beta_0$ are also examined. Finally, we identify similarities between $B_y=0$ and $B_y \neq 0$ cases in early times.

A. Coplanar case ($B_y=0$)

Figure 2 shows spatial profiles of $\rho$, $T$, $P$, $B$, $V_y$, $B_y$, $V_x$, and $J_x$ at $t=500$ (dash-dotted lines), 2000 (dotted lines), and 5000 (solid lines). The initial current sheet is located at $z=0$ (the center of simulation domain), and the initial parameters are $\beta_0=0.04$, $\theta=70^\circ$, and $\phi_0=0$. In the early time with $t=500$, a pair of fast rarefaction waves is formed at $|z| \sim 300-500$ and a pair of slow shocks is formed at $|z| < 100$. As a result of the fast expansion waves (FE), the plasma density and magnetic field are reduced from the ambient values. Large jumps in $\rho$, $P$, $B$, $V_x$, and $B_x$ occur across slow shocks. In this coplanar case, $B_y$ remains null and $B_x$ downstream of slow shocks is zero, indicating that slow shocks are switch-off shocks. The case with $B_y=0$ is a singular case in which TDISs are not present. In a later time with $t=2000$, fast expansion waves have propagated out of the boundary, while slow shocks keep propagating outward.

At $t=5000$, the slow shock on the right-hand side is denoted by “a.” There are several features in the temporal evolution of slow shock: (1) the downstream plasma density decreases with time, (2) the downstream temperature increases with time, and (3) the downstream plasma pressure remains relatively constant in time. These basic temporal characteristics are also maintained for the $B_y \neq 0$ case to be presented later.

B. Noncoplanar case ($B_y \neq 0$)

By adding a constant $B_y$ in an initial state as that in Fig. 2, Fig. 3 shows the results at $t=500$ (dash-dotted lines), 2000 (dotted lines), and 5000 (solid lines) with an additional appearance of TDISs. Lin and Lee34 also reported that TDISs are generated in $B_y \neq 0$ cases. The initial parameters are $\beta_0=0.04$, $\theta=70^\circ$, and $\phi_0=15^\circ$. Because fast expansion waves have propagated out of simulation domain, the plasma density and magnetic field at $t=2000$ and 5000 on the two sides are reduced from ambient values. Compared to the $B_y=0$ case, some features for the $B_y \neq 0$ case are: (1) the speed of slow shocks decreases, (2) the jump of plasma density across slow shock increases, and (3) jumps of pressure, temperature, and magnetic field across slow shock decrease.

The slow shock denoted by symbol “a” is located at $z \sim 600$ at $t=5000$ (solid lines). The rotation of magnetic field at $z \sim 700-900$ shows the existence of TDIS. It can be identified by the current density $J_z = -dB_y/dz$. In the $B_y$ profile, the indicator “b” denotes the position of maximum $|J_z|$, the center of TDIS structure. Because TDIS rotates the direction of tangential magnetic field, the downstream $B_y$ component increases gradually as the field rotation angle $\Delta \phi$ in-
creases across TDIS. On the other hand, the \( B_y \) component decreases across slow shock and keeps a nonzero constant downstream of slow shock. Note that the slow shocks are not switch-off shocks. Due to reduction of antiparallel magnetic field component \( (B_y) \) and sharing of the additional TDIS pair on magnetic energy conversion, slow shocks become weaker than those without TDISs. Other important features in the structure of TDISs are: (1) the downstream plasma temperature, pressure and density across TDIS increase, (2) the rotation of tangential magnetic field across TDIS develops with time and gradually reaches a final state, and (3) across the final TDIS the downstream magnetic intensity decreases slightly and \( B_y \) decreases to zero.

Figure 4 plots time evolutions of magnetic field

![Hodogram of tangential magnetic field](image1)

![Hodogram of tangential velocity](image2)

**FIG. 3.** Spatial profiles of \( \rho, T, P, B_y, V_x, V_y, B_x, V_z, \) and \( J_x \) at \( t=500 \) (dot-dashed lines), 2000 (dotted lines), and 5000 (solid lines) for the cases with \( \phi_s=15^\circ, \beta_s=0.04, \) and \( \theta=70^\circ \). Here, “a” indicates slow shock and “b” indicates TDIS.

**FIG. 4.** The magnetic field hodograms and velocity hodograms for the case with \( \phi_s=15^\circ, \beta_s=0.04, \) and \( \theta=70^\circ \) at different times. Here, “1-2” indicates the fast expansion wave, “b” indicates TDIS, “a” indicates slow shock, “3” denotes downstream of TDIS and “4” denotes downstream of slow shock.
hodogram and velocity hodogram for the $B_z \neq 0$ case in Fig. 3. For both hodograms at $t=2000$, state 1 to 2 indicates the fast expansion wave (FE), “a” indicates the magnitude jump (from state 3 to 4) for slow shock, and “b” indicates directional rotation (from state 2 to 3) for TDIS. The final upstream and downstream magnetic fields of slow shocks at $t=5000$ are coplanar with the normal vector $(\mathbf{n})$, while the upstream (downstream) angle ($\phi$) of tangential magnetic field of TDIS is $15^\circ$ ($90^\circ$). After switching off of the $B_z$ component across TDIS, the slow shock reduces the $B_z$ component of magnetic field.

As seen in the magnetic field hodogram, FE (from upstream state 1 to downstream state 2), TDIS (from 2 to 3), and SS (from 3 to 4) are formed in the resulting current sheet evolution. In earlier times ($t=250,500,1000,2000$) the upstream and downstream magnetic fields of slow shocks and shock normal are noncoplanar, and slow shocks keep evolving with time. These slow shocks can be called “quasi-slow shocks.” There exists a central wedge in magnetic hodograms for $t<2000$. The rotation of tangential magnetic field across TDIS evolves to close this wedge. At $t=5000$, the TDIS has reached a final state, in which TDIS switches off the upstream $B_z$ component, and leads to a coplanar slow shock with upstream and downstream magnetic fields lying in the $y$-$z$ plane. We should note that the magnetic fields associated with slow shock are in the $x$-$z$ plane for the $B_z=0$ case.

From the velocity hodograms at earlier times ($t<2000$), we find that the TDIS leads to a graduate increase of both downstream tangential velocity components ($V_x$ and $V_y$). At a later state ($t=5000$), the $V_y$ component downstream of TDIS has reached a maximum value and at this time SS only reduces the $V_y$ component to zero, keeping $V_z$ constant. This situation is very different from the $B_z=0$ case. For the $B_z=0$ case without the presence of TDIS, SS only increases $V_y$, while the $V_y$ component always remains zero (see Fig. 2).

Figure 5(a) plots the rotational angle ($\Delta \phi$) of tangential magnetic field across TDIS as a function of time for the case in Fig. 3. In earlier times ($t<2000$), $\Delta \phi$ increases quickly, and then slows down for $t>2000$. Finally, the rotation angle $\Delta \phi$ reaches the final value $\Delta \phi_{final}=75^\circ$. As TDIS reaches the final state switching off the $B_z$ component, the wedge in the hodograms of Fig. 4 is closed ($t=5000$). We use the symbol $t_c$ to indicate the time when $\Delta \phi$ reaches 90% of final rotational angle ($\Delta \phi=\Delta \phi_{final} \times 90\%$). In this case the characteristic time is $t_c=1732$ and the rotation angle $\Delta \phi=\Delta \phi_{final} \times 90\%=75^\circ \times 90\%=67.5^\circ$. The rotational angle $\Delta \phi$ across TDIS develops with time gradually, and the final angle obeys $\Delta \phi_{final}=90^\circ-\phi_c$.

Figure 5(b) shows the ratio $P_z/P_0$ as a function of time for $\phi_c=0^\circ$ and $\phi_c=15^\circ$, where $P_z$ is the downstream pressure of slow shock and $P_0$ is the upstream pressure far away from the initial current sheet. For $\phi_c=15^\circ$, the strength of slow shock quickly reaches its reduced final value, while it takes a much larger time for TDIS to reach its final state.

III. OTHER FEATURES OF CURRENT SHEET EVOLUTION

A. Dependence of current sheet evolution on the initial rotation angle $\phi_c$

Figure 6 plots magnetic hodograms at final stage for different values of $B_z$ or $\phi_c$. The other initial parameters for these cases are the same as in Fig. 3. Since a pair of fast expansion waves (FE) has propagated out of the simulation boundary, we cannot see FE and the sign “1” in the hodograms. Notice that $\phi_c=0^\circ$ is a singular case without the presence of TDIS, but any small value of initial $B_z$ magnetic field will trigger generation of TDISs. With an increasing $\phi_c$, the final rotational angle $\Delta \phi_{final}$ across TDIS decreases, and the $B_z$ jump across slow shock from 3 to 4 decreases. Thus, the slow shock transfers less magnetic energy to kinetic/thermal energy.

We further analyze the time evolution of $\Delta \phi$ across TDIS for $\phi_c=5^\circ$, $10^\circ$, $15^\circ$, $30^\circ$, $45^\circ$, $60^\circ$, $75^\circ$, $85^\circ$, and $89^\circ$.
in Fig. 7. The initial parameters for these cases are the same as in Fig. 6. The cases with \( \phi_\infty = 5^\circ, 10^\circ, \) and \( 15^\circ \) are plotted with solid lines, cases with \( \phi_\infty = 30^\circ, 45^\circ, \) and \( 60^\circ \) are plotted with dashed lines, and cases with \( \phi_\infty = 75^\circ, 85^\circ, \) and \( 89^\circ \) are plotted with dotted lines. The results show that it takes a longer time to reach the final state for a smaller \( \phi_\infty. \) At \( t=5000, \) the rotational angle \( \Delta \phi \) across TDIS in cases with \( \phi_\infty = 5^\circ \) and \( \phi_\infty = 10^\circ \) has not reached final values. Figure 7 also shows that the final rotational angle \( \Delta \phi_{\text{final}} \) increases with decreasing \( \phi_\infty. \) The \( \phi_\infty = 0 \) case is a singular case without TDIS.

Figure 8 plots time \( t_c \) for the rotational angle of magnetic field across TDIS to reach 90% of its final value \( \Delta \phi_{\text{final}} \) as a function of \( \phi_\infty. \) The initial parameters for these cases are \( \beta_\infty = 0.04 \) and \( \theta = 70^\circ. \) The results show that with increasing \( \beta_\infty, \) TDIS rotates with a small \( \Delta \phi \) and quickly reaches its final state. Figure 9 shows that the final rotation angle \( \Delta \phi_{\text{final}} \) across TDIS is related to \( \phi_\infty \) by

\[
\Delta \phi_{\text{final}} = 90^\circ - \phi_\infty.
\]

Note that \( \phi_\infty = 0 \) is a singular case with \( \Delta \phi = 0 \) and no \( B_y \) component is needed to keep the slow shock coplanar. Any small finite value of initial \( B_y \) guide field will trigger generation of TDIs to have final coplanar slow shocks in the \( y-z \) plane. The linear relation in Fig. 9 is valid anywhere for any finite value of \( \phi_\infty. \) The \( \phi_\infty = 0 \) case is a singular case with \( \Delta \phi_{\text{final}} = 0. \)

B. Dependence on plasma beta \( \beta_\infty \)

To examine effects of background plasma beta \( (\beta_\infty) \) on shock evolution, Fig. 11 shows simulation results for \( \beta_\infty = 0.04, 0.2, 0.5, \) and 1.0 at \( t=1500. \) In all cases, we have set \( \phi_\infty = 15^\circ \) and \( \theta = 70^\circ. \) Figure 11 shows that with increasing
the pressure and temperature downstream of slow shocks increase, while the plasma density decreases, (2) fast expansion waves become stronger, and (3) propagation speeds of slow shocks and TDISs increase. Figure 12 shows the rotational angle $\Delta \phi$ of magnetic field across TDIS as a function of time for five cases with $\phi_s=15^\circ$, $\theta=70^\circ$, and $\beta_s=0.01, 0.04, 0.2, 0.5, 0.8$, and $1.0$. The time for $\Delta \phi$ to reach the final value $\Delta \phi_{\text{final}}$ for TDIS increases with increasing $\beta_s$, but the relation, $\Delta \phi_{\text{final}}=90^\circ-\phi_s$, still holds.

We also plot the ratio of tangential magnetic energy reduction, $(B_{\text{t3}}^2-B_{\text{t4}}^2)/B_{\text{t3}}^2$, and Alfvén Mach number ($M_\Lambda=V_{S\Lambda}/V_A$) across slow shock as a function of $\beta_s$ in Fig. 13. The initial parameters for these cases are $\phi_s=45^\circ$, $\theta=70^\circ$. With increasing $\beta_s$, both the ratio $(B_{\text{t3}}^2-B_{\text{t4}}^2)/B_{\text{t3}}^2$ and Mach number $M_\Lambda$ increase.

Figure 14 shows magnetic field hodograms for cases with $\phi_s=15^\circ$, $45^\circ$, and $60^\circ$ and plasma beta $\beta_s=0.04, 0.5, 1.0$, and $5.0$ at $t=5000$. The initial normal angle are $\theta=70^\circ$. 

**FIG. 10.** The ratio of tangential magnetic energy reduction, $(B_{\text{t3}}^2-B_{\text{t4}}^2)/B_{\text{t3}}^2$, and the Alfvén Mach number ($M_\Lambda=V_{S\Lambda}/V_A$) across slow shock as a function of $\phi_s$. The other initial parameters for these cases are $\beta_s=0.04$ and $\theta=70^\circ$.

**FIG. 11.** Spatial profiles of $\rho$, $T$, $P$, $B_x$, $B_y$, $V_x$, and $V_z$ at $t=2000$ for four cases with $\beta_s=0.04$, $0.2$, $0.5$, and $1.0$. The other initial parameters for this case are $\phi_s=15^\circ$ and $\theta=70^\circ$.

**FIG. 12.** The rotational angle ($\Delta \phi$) of tangential magnetic field across TDIS as a function of $t$ for five cases with $\phi_s=15^\circ$, $\theta=70^\circ$, and $\beta_s=0.04, 0.2, 0.5, 0.8$, and $1.0$.

**FIG. 14.** Magnetic field hodograms for cases with $\phi_s=15^\circ$, $45^\circ$, and $60^\circ$ and plasma beta $\beta_s=0.04, 0.5, 1.0$, and $5.0$ at $t=5000$. The initial normal angle are $\theta=70^\circ$. 

and the Alfvén Mach number.

Figure 14 shows that (1) a larger $\phi_c$ leads to a smaller $\Delta \phi$ across TDIS, weaker magnetic jump across slow shock, and shorter time to reach final state as discussed earlier, and (2) a larger $\beta_c$ leads to a stronger magnetic jump across slow shock and a longer time to reach final state.

C. Similarities between $B_y=0$ and $B_y \neq 0$ cases in early times

Figure 15 shows spatial profiles of $\rho$, $V_x$, $B_x$, $V_y$, and $B_y$ for the coplanar case with $\phi_c=0^\circ$ ($B_y=0$), $\beta_c=0.04$, and $\theta=70^\circ$ (dotted lines) and noncoplanar case with $\phi_c=5^\circ$ ($B_y \neq 0$), $\beta_c=0.04$, and $\theta=70^\circ$ (solid lines). It is interesting to observe that the spatial profiles of $\rho$, $V_x$, and $B_x$ are nearly identical for the coplanar case ($B_y=0$) and noncoplanar case ($B_y \neq 0$) for $t=500$, 1000, and 2000, and even for $t=5000$. Furthermore, perturbations of $V_y$ and $B_y$ are very small in early times and grow with time, while large variations in $\rho$, $V_x$, and $V_y$ are already present in early times. Therefore, we may suggest that the early current sheet evolutions for the $B_y=0$ and small $B_y$ cases are very similar. Due to time evolution and growth of TDISs in the noncoplanar case, spatial profiles of $\rho$, $V_x$, $V_y$, $B_x$, and $B_y$ become very different from those in the coplanar case in later times. It is also interesting to observe that $\rho$, $V_x$, and $B_y$ profiles in Fig. 2 ($\phi_c=0^\circ$ case) and in Fig. 3 ($\phi_c=15^\circ$ case) are also very similar.

The similarities of plasma density, magnetic field, and velocity evolutions between the $B_y=0$ and small $B_y$ cases in early times have an interesting implication for magnetic reconnection at the dayside magnetopause and nightside magnetotail current sheet. The results imply that the structure of reconnection layer, which consists of shocks and discontinuities, will be very similar in the region not far away from the reconnection X line for the $B_y=0$ and small $B_y$ cases.

IV. SUMMARY AND DISCUSSION

A simulation based on MHD formulations is performed to examine effects of guide field ($B_y$), and $\beta_c$ on the evolution of a current sheet. The results can be applied to magnetic reconnection in solar flares, corona heating, geomagnetic substorms, and tokamaks. We summarize our main results as follows:

1. For the $B_y=0$ case, a pair of slow shocks is formed. For this coplanar case, $B_x$ remains null and $B_y$, downstream of slow shocks is zero, indicating that the slow shocks are switch-off shocks. For the $B_y \neq 0$ cases, a pair of slow shocks and an additional pair of TDISs are present in the reconnection layer. TDISs rotate the direction of tangential magnetic field, so that the downstream $B_x$ component gradually increases. The $B_x$ component then decreases across slow shocks and keeps a nonzero constant downstream of slow shocks. For this noncoplanar case, the slow shocks are not switch-off shocks.

2. In the case with $B_y \neq 0$ ($\phi_c \neq 0$), the upstream and downstream magnetic fields of slow shocks are noncoplanar in earlier times, and keep evolving with time, until the strength of these slow shocks reaches their final value. These transient slow shocks can be called “quasi-slow shocks.” The rotational angle $\Delta \phi$ of tangential magnetic field across TDISs develops with time gradually, and the final angle obeys $\Delta \phi_{final}=90^\circ-\phi_c$. In the final state, TDIS switches off the upstream $B_x$ component, and leads to a co
planar slow shock with upstream and downstream magnetic fields lying in the y-z plane.

(3) Any small finite value of initial $B_2$ guide field triggers generation of TDISs, while $\phi_0=0$ is a singular case without TDISs. It needs a very long time for the simulation with a very small $\phi_0$ to reach the final state.

(4) As $\phi_0$ increases or plasma beta $\beta_v$ decreases, the Alfvén Mach number associated with slow shocks decreases, and the amount of magnetic energy converted into kinetic energy also decreases.

(5) The early current sheet evolutions for the $B_2=0$ and small $B_2$ cases are very similar. The spatial profiles of $\rho$, $V_x$, and $B_x$ in both cases are nearly identical. For the small $B_2$ case, perturbations of $V_y$ and $B_y$ are very small in early times and grow with time, while large variations in $\rho$, $V_x$, and $V_y$ are already present in early times as in the $B_2=0$ case. Due to time evolution and growth of TDISs in the noncoplanar case ($B_1 \neq 0$), the spatial profiles of $\rho$, $V_x$, $V_y$, $B_x$, and $B_y$ become very different from those in the coplanar case.

**ACKNOWLEDGMENTS**

This work was supported by grants from the National Science Council in Taiwan (NSC 94-2111-M-008-034 and NSC 95-2111-M-008-037) to the National Central University and a grant to the Earth Dynamic System Research Center from National Cheng Kung University.

---