Dominance of second Bessel peak in relativistic electromagnetic ion cyclotron instabilities driven by fusion-produced fast ions

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Relativistic electromagnetic ion cyclotron instabilities driven by fusion-produced fast ions in magnetized plasmas can have two peaks in their growth rate spectrum. The wave numbers of these two peaks are close to the first and second peaks, respectively, of the Bessel function that is in the resonance driving term. The driving of the second Bessel and growth rate peak occurring at a higher wave number is weaker than that of the first peak. Surprisingly, as in contrast to conventional wisdom, the second peak can dominate near the instability threshold. For the higher energy of fusion-produced fast ion such as 14.7 MeV, the slow ion temperature is required to be higher for overcoming the threshold to drive a cubic instability, which is determined by an Alfvénic condition. This cubic instability is due to the coupling of the first-order slow ion resonance and second-order fast ion resonance. This finite temperature effect is on the slow ion resonance and increases with wave number and thus the threshold is first satisfied near the second peak. Therefore, the second peak appears earlier in the instability spectrum and dominates near the threshold. The cubic instability has a much larger frequency mismatch than a coupled quadratic instability; a larger frequency mismatch indicates more fast ion energy to loss before the nonlinear saturation of the instability. When the slow ion temperature or density is about twice that of the threshold, the second peak has transited from the cubic to the coupled quadratic instability while the first peak remains as the cubic instability, in contrast to the previous 3.02 MeV proton case. © 2007 American Institute of Physics. [DOI: 10.1063/1.2769290]

I. INTRODUCTION

Ions produced in fusion reactions are energetic. The kinetic energy of the proton produced from deuterium and deuterium reaction is 3.02 MeV while that from deuterion and helium 3 reaction is 14.7 MeV. The kinetic energy of the fast ions is the only source for direct reheating the burning plasma to maintain thermonuclear fusion reactions. Therefore, the instability driven by the fast ions is an important issue in fusion reactors and an active research area in plasma physics. In fact, cyclotron emissions by fast ions are experimentally measured in the Joint European Tokamak and the Test Fusion Tokamak Reactor. Electromagnetic ion cyclotron wave can propagate and be detected outside plasmas, and thus its instabilities are deserved to be studied.

The Lorentz factor of the fast ions is very close to unity. The importance of relativativity for driving ion cyclotron instabilities is discovered for only one decade, while it has been well known for five decades for relativistic electron cyclotron instabilities. The study of relativistic ion cyclotron instability was focused on the electrostatic modes. The electromagnetic relativistic ion cyclotron instability was just studied recently.

Interesting physics such as the Alfvénic behavior and the instability transition were discovered for the electromagnetic relativistic ion cyclotron instabilities. An Alfvénic condition requires the ratio of slow ion temperature and fast ion energy high enough for a cubic instability to occur, and thus is an important issue for applications in devices. The instability transition is an interesting novel phenomenon. The transition of plasma instabilities usually requires the dramatic change of plasma conditions; e.g., the velocity distribution of plasmas has changed from beam to thermal when two-stream-instability transits to Landau instability. However, when the electromagnetic ion cyclotron instability transits from the cubic instability to a coupled quadratic instability, the plasma characteristics such as distribution function remain the same and only plasma parameters change. Cubic instability and quadratic instability usually have different physics. For the relativistic ion cyclotron instabilities, the order of resonances involved in driving instability and the resultant frequency mismatches are different. Thus, they have sharp-contrasted consequences. For the relativistic electrostatic ion cyclotron instabilities, the cubic instability causes fast protons to be thermalized and lose a significant fraction of their kinetic energy to wave and then to slow deuterons while the coupled quadratic instability...
selectively broadens the energy distribution of fast alpha particles.\cite{footnote:energy-broadening}

The driving of the cubic instability is from the second order resonance of fast ions that is coupled with a first order resonance of slow ions and the resultant relativistic bunching in the gyrophase. This driving term depends on the Bessel function of the first kind with the order of the resonant harmonic number. For the first harmonic, it has a larger driving at the first peak near \(k \rho_f = 4\) (where \(k\) is the wave number and \(\rho_f\) is the largest fast ion Larmor radius corresponding to the birth energy) and a weaker driving at the second peak near \(k \rho_f = 7\). For the previously studied case of 3.02 MeV,\cite{footnote:3.02-MeV} the first peak satisfies the Alfvénic condition first when the slow ion temperature or density is raised higher than the threshold. Also, this first peak transits from the cubic instability to the coupled quadratic instability when the slow ion temperature or density has been raised to about twice the threshold.

In this paper, we study the surprising novel physics for the second peak with a weaker driving to dominate in the case of higher fast ion energy such as 14.7 MeV. With higher fast ion energy, the slow ion temperature is required to be higher for overcoming the threshold so that the finite slow ion temperature effect becomes important. This effect causes the interesting new physics. A homogeneous plasma is considered here in order to display the basic physics phenomenon, not known so far, although for actual confinement systems the impact of inhomogeneities\cite{footnote:inhomogeneities} requires consideration.

The analysis of the instabilities including the finite temperature effect will be studied in the next section. Section III explains the results of the numerical studies of the general dispersion relation derived from kinetic theory. The summary is in Sec. IV.

\section{Analysis of Alfvénic Condition and Cubic Instability Including Second Order Finite Slow Ion Temperature Effect}

We consider a fast ion species in a uniform plasma under an external magnetic field along the \(z\) direction (i.e., \(\vec{B} = B \hat{z}\)). An electromagnetic wave propagates along the \(x\) direction with the wave vector \(k = k \hat{x}\) and its electric field is polarized in the \(y\) direction. The dispersion relation has been derived\cite{footnote:dispersion} from the kinetic theory in the form of a cubic algebraic equation of the difference between the wave frequency \(\omega\) and the harmonic fast ion cyclotron frequency \(\ell \omega_{c,f}\).

As compared with the 3.02 MeV proton previously studied,\cite{footnote:3.02-MeV} the 14.7 MeV proton has a larger Lorentz factor, even if it is still very close to unity. Thus, the frequency mismatch between harmonic fast and slow ion cyclotron becomes larger. Also, as discussed later, higher fast ion energy will increase the threshold of the slow ion temperature for satisfying the Alfvénic condition. Thus, the resultant slow ion gyroradius does not become very small, so that more terms have to be kept in the expansion of \(h_\ell(\lambda)\) over \(\lambda\) in the resonant slow ion dielectric.\cite{footnote:resonant-slow-ion-dielectric} Therefore,

\[
h_\ell(\lambda) = \left[ \frac{s^3 \lambda^{t-1}}{s! 12^{t}} + \frac{s^2 \lambda^{t+1}}{(s+1)! 12^{t+1}} + \frac{\lambda^{t+1}}{s! 12^{t+1}} - \frac{\lambda^t}{(s-1)! 12^{t-1}} \right] \times \left( 1 - \lambda + \frac{\lambda^2}{2} \right),
\]

where \(\lambda = \left( k^2 T / m \Omega^2 \right) = k^2 \rho^2, T\) is the slow ion temperature, \(s\) is the harmonic number of the slow ion, \(m\) is the rest mass, \(\rho = v_\perp / \omega_c\) is the gyroradius (or Larmor radius), \(v_\perp\) is the perpendicular gyrovelocity, \(\Omega\) is the nonrelativistic cyclotron frequency, and the subscript \(\ell\) (for slow fast ion).

The coupling of the second order fast ion resonance and the first order slow ion resonance is needed for the cubic instability. The condition in Ref. \cite{footnote:condition} can be rewritten as

\[
0 = \eta = \frac{PA}{\ell \omega_{c,f}} \equiv \frac{A \Delta + D}{\ell \omega_{c,f}} = \frac{1}{\ell \omega_{c,f}} \left[ \frac{k^2 \rho^2 \omega_{c,f}}{2} + \delta \gamma \sum_{s=1, s \neq 1} a_s \omega_{ps} - \frac{\omega_{ps}^2}{1 + a_m} h_\ell(\lambda) \right],
\]

where

\[
P = \Delta + \frac{B + D}{A}.
\]

\[A = 1 - \frac{k^2 c^2}{\omega^2} - \frac{\omega_{ps}^2}{\omega_{ce}^2} - \sum_{s=1, s \neq 1} \frac{2 \omega_{ms}^2}{\ell^2 \omega_{c,f}^2 - \Omega^2_{cs}} h_1,\]

\[B = \frac{\omega_{ps}^2}{2 \omega_{c,f}^2} \left[ (\ell \omega_{c,f}^2 - \Omega^2_{cs}) - 1 / \ell^2 (k^2 \rho^2) \ell \omega_{c,f}^2 \right],\]

\[D = \frac{\omega_{ps}^2}{\ell \omega_{c,f}} h_\ell(\lambda),\]

\[\Delta = \ell \omega_{c,f} - s \omega_{cs} \equiv \ell \omega_{c,f} \left( 1 - \frac{s \Omega^2_{cs}}{\ell \Omega^2_{c,f}} \right) \approx \ell \omega_{c,f},\]

the factor \(\eta\) is kept as a correction term for the small deviation away from zero; the coefficient \(P\) determines the characteristics of the instabilities with the factor \(\Delta\) being the frequency mismatch between the harmonic cyclotron frequencies of fast and slow ions; the coefficient \(A\) is the nonresonant plasma dielectric representing the inertia of the plasma; the coefficient \(D\) is the slow ion first-order resonance; \(a_m = (\partial m / \partial n) / \delta \gamma, \delta \gamma = \gamma - 1, \delta m = 1 - m / m_p\), defined as the normalized mass defect per nucleon of the ions, \(N\) the nucleon number of the ions, and \(m_p\) the proton rest mass; \(\rho_0\) is the maximum fast ion Larmor radius; \(a_n = (1 - \Omega^2_{cs} / \ell^2 \omega_{c,f}^2)^{-1} \); \(c\) is the speed of light; \(\omega\) is the wave frequency; \(\omega_p\) is the plasma frequency; \(\omega_c\) is the cyclotron frequency; the subscript \(e\) is for electron; \(J_n(\lambda)\) is the Bessel function of first kind of order \(n\) with the parameter \(k \rho\).

For fast proton in thermal deuterium plasma, the second deuteron harmonic resonants with the first proton harmonic. For \(s = 2,\)
Taking the derivative of \( h_2(\lambda) \), we obtain the higher-order correction term for finite \( \lambda \). The finite slow ion temperature (or warm plasma) effect reduces the magnitude of \( h_2 \) and thus the resonant inertia of the slow ion. This reduction increases with the wave number. Since the peak of the coefficient \( P \) is determined by the balance of the nonresonant and the resonant slow ion inertias, it is moved to higher wave number. This will have profound effects on the instability peaks.

By substituting Eq. (8) into Eq. (2), we have

\[
\frac{\xi \omega_{ps}^2}{2(1 + a_m)} \lambda^2 + \left[ \frac{\omega_{ps}^2}{2} \frac{\rho_0}{\rho_s} - \frac{\omega_{ps}^2}{2(1 + a_m)} \right] \lambda + \left( \delta \gamma a_s \omega_{ps}^2 - \frac{\omega_{ps}^2}{1 + a_m} \eta \right) = 0. \tag{9}
\]

The inequality for two real \( \lambda \) roots yields the Alfvénic condition, including the \( \eta \) correction term, for instability to exist as

\[
\frac{T}{E_f} > \frac{1 + a_m}{1 - 2 \xi (1 + a_m) \left( \delta \gamma a_s - \frac{\omega_{ps}^2}{\omega_{ps}^2 + 1 + a_m} \right)} \left[ 1 + \frac{B^2}{2 \pi m_c e^2 n_s} \right], \tag{10}
\]

where \( E_f \) is the fast ion energy, and \( n_s \) is the slow ion density. The right-hand side can also be written as having the square of the Alfvén velocity \( c_A = \Omega_A/\omega_{ps} = B/(4\pi n_c m_e)^{1/2} \). Without the finite \( \lambda \) correction term \( \xi \), this inequality recovers the previous result. \(^{25}\) We may define the ratio of the plasma pressure and the magnetic pressure as

\[
\beta = \frac{n_c T}{B^2/18\pi}, \tag{11}
\]

so that the Alfvénic condition becomes

\[
\beta > \frac{1 + a_m}{1 - 2 \xi (1 + a_m) \left( \delta \gamma a_s - \frac{\omega_{ps}^2}{\omega_{ps}^2 + 1 + a_m} \right)} \left[ 1 + \frac{4}{m_c e^2 n_s} \right] B_f. \tag{12}
\]

The location of the coefficient \( P \) peak can be found by taking the derivative of \( P \) by \( k p_0 \) and setting to zero. For \( s=2 \),

\[
P \approx -\frac{\omega_{ps}^2}{2} \left( 1 - \xi \lambda \right) + \frac{k^2 c^2}{\omega_{ps}^2} \left( \frac{\omega_{ps}^2}{\omega_{ps}^2} - \frac{1}{\omega_{cf}^2} \right). \tag{13}
\]

The peak is located at

\[
k p_0 = \frac{8\pi \gamma m_f n_T}{m_i T} B^2 \left[ -1 + \frac{1}{4\pi n_c T} \frac{B^2}{\xi a_s} \right]^{1/2}, \tag{14}
\]

or in a format with \( \beta \) as

\[
k p_0 = \frac{a_s \gamma m_f n_T}{m_i T} \beta \left[ -1 + \frac{2}{\beta a_s} \right]^{1/2}. \tag{15}
\]

The wave number location of the coefficient \( P \) peak and the unstable spectral peak increases with the fast ion energy (or the slow ion temperature while the ratio of the fast ion energy and the slow ion temperature is fixed).

### III. Numerical Studies

For a broader applicability and with fewer assumptions, we will numerically study the general dispersion relation given in Eq. (4) of Ref. 25 for the electromagnetic relativistic ion cyclotron instabilities. The numerical results will be compared with the analytical results given in the previous section. Interesting physics will be illuminated.

The energy of the proton produced by D-\(^{3}\)He fusion reaction is 14.7 MeV in contrast to 3.02 MeV from the D-D reaction studied in Ref. 25. A negative frequency mismatch is still required for both the cubic and coupled quadratic instabilities. Proton has a nucleon with one positive charge and its rest mass is 1836.15 \( m_p \). The normalized mass deficit per nucleon of proton is zero (i.e., \( \delta m_p=0 \)) because we define proton mass to be unity. Deuteron has two nucleons and one positive charge with its rest mass being 3670.48 \( m_p \) so that the normalized mass deficit per nucleon of deuteron is \( \delta m_d=0.000496 \). The fast protons with the energy of 14.7 MeV has a Lorentz factor of \( \gamma_p=51.8 \) keV. As discussed later, the threshold deuteron temperature is 28.9 keV. However, the threshold condition including the finite slow ion temperature (or warm plasma) effect in Eq. (10) indicates that the threshold deuteron temperature is \( T=51.8 \) keV. As discussed earlier, the threshold temperature obtained numerically is 48.5 keV. Thus, we choose the deuteron temperature of 55 keV as our typical case. The corresponding plasma parameters are the deuteron plasma frequency \( \omega_{pd}=6.58 \times 10^5 \) rad/s, the deuteron cyclo-
electron frequency $\omega_{cd}=1.44 \times 10^8$ rad/s, the fast proton plasma frequency $\omega_{pp}=9.24 \times 10^7$ rad/s, the relativistic fast proton cyclotron frequency $\omega_{f}=2.83 \times 10^8$ rad/s, the electron plasma frequency $\omega_{pe}=3.99 \times 10^{11}$ rad/s, the electron cyclotron frequency $\omega_{ce}=5.28 \times 10^{11}$ rad/s, the maximum gyroradius of the fast protons $r_0=18.5$ cm, and the gyroradius of a deuteron with the perpendicular thermal velocity $v_0=1.13$ cm. Also, $d_m=0.0316$ and $a_s=1.35$ for $\epsilon=1$. Since the deuteron cyclotron frequency is about half of the proton cyclotron frequency, the resonant cyclotron harmonic number of the deuterons is twice that of the fast protons.

Figure 1 shows the growth rate and real frequency of the electromagnetic ion cyclotron frequency as a function of the wave number for the typical case; the wave is resonant with first proton harmonic and second deuteron harmonic. Both are normalized by the fast proton cyclotron frequency, while the normalized real frequency is also deducted by the harmonic number of the deuterons is twice that of the fast protons.

The normalized growth rate is located at the normalized wave number of 7.2 and there is no instability at $k\rho_0=4$. The relativistic cyclotron instability is driven by the gyrophase bunching of harmonic cyclotron motion, that results in the second order resonance. This provides the dispersion relation $P$ an extra term $R$ (or $C$ in different format of dispersion relation) for the electromagnetic ion cyclotron instability. As analyzed in the previous section, this instability driving term is proportional to the square of the derivative of the Bessel function of first kind with its order of the harmonic cyclotron number. For first harmonic, the strongest peak is located near $k\rho_0=4$ while there is a weaker peak near $k\rho_0=7$. Thus, it is interesting to note that, as against conventional wisdom, the instability peaks where the instability driver is weaker and it is stable at where the driving is the strongest.

To understand this anomalous result, we have to check the instability condition. For the cubic instability, the coefficient of the harmonic cyclotron resonance of slow ions has to be just large enough to overcome the mismatch between the fast and slow ions' harmonic cyclotron frequency and the inertia of the nonresonant plasmas; that is, the coefficient $P$ is required to be small. Figure 2 shows the coefficient $P$ in the previous 3.02 MeV and current 14.7 MeV cases. For the previous case, when the slow ion temperature is 8 keV that is just over the threshold, the peak of the coefficient $P/\omega_{cf}$ (the growth rate) is located at $k\rho_0=5.25(4.24)$ with a small negative value of $-0.000353$ whose absolute value has been reduced to be close to the normalized difference of the wave frequency and the fast ion cyclotron frequency that is $0.000306(0.000454)$. The threshold of the slow ion temperature calculated by the Alfvénic condition [Eq. (31) of Ref. 25] is 6.64. When the slow ion temperature is risen to 10 keV, the peak of the coefficient $P/\omega_{cf}$ (the growth rate) moves to a lower wave number of $k\rho_0=4.9(3.68)$ and its corresponding value becomes a small positive (negative) number of $0.00036(-0.000188)$; the absolute value is smaller than the normalized frequency difference of 0.000392. The coefficient $P$ is zero at $k\rho_0=3.23$ and 7.2; they are close to where the first and second growth rate peaks are, respectively. For the 14.7 MeV case studied here, the frequency mismatch is larger because of the larger Lorentz factor of the proton due to higher kinetic energy. If the same equation of the Alfvénic condition without considering the finite slow ion temperature effect is used, the threshold of slow ion temperature is increased with the fast ion kinetic energy to be 28.9 keV. However, the numerical calculation indicates that the threshold is 48.5 keV. The higher threshold is due to the finite slow ion temperature and $P$ effects as indicated in Eq. (10) that predicts 51.8 keV. The slow ion temperature of 50 keV is just high enough to overcome the instability con-
dition. The peak value of the coefficient $P/\omega_{ci}$ located at $k_{\rho_0} = 6.94$ is $-0.0015$ whose absolute value is close to the absolute value of the normalized frequency mismatch between the wave and the ion cyclotron that is 0.0012. Thus, in addition to a higher threshold of slow ion temperature, the location of the coefficient $P$ peak has moved to a higher wave number at $k_{\rho_0} = 6.94$, that is near the second peak of the driving $R$ or $C$ term. As revealed by Eq. (15), the wave number location of the coefficient $P$ peak increases with the $\beta$ value. When the slow ion temperature increases further, the location of the coefficient $P$ peak moves to lower wave number with its value toward zero. Because the Alfvénic condition is satisfied first at a higher wave number near the second peak of the driving term, the second growth rate peak can appear first and dominate the Alfvénic behavior near the threshold.

B. Slow ion temperature

Figure 3 shows the growth rate spectrums as a function of the wave number for different slow ion temperatures. When the slow ion temperature is 50 keV that is just slightly higher than the threshold, a single growth rate peak appears at the wave number of $k_{\rho_0} = 7.2$ near the second driving peak. As the slow ion temperature is increased, this second growth rate peak is increased at first and then decreased. Also the first growth rate peak near $k_{\rho_0} = 4$ appears when the slow ion temperature is higher. These results can be understood by the coefficient $P$ shown in Fig. 4. The corresponding wave number of the coefficient $P$ peak is lower with the increase of the slow ion temperature so that it becomes more and more favorable for the first growth rate peak. There are two $P = 0$ locations at higher slow ion temperature. As the slow ion temperature is increased, the first $P = 0$ location moves to lower wave number so that the instability at first growth rate peak is excited; however, the third growth rate peak as seen before $^{25}$ has not appeared yet for the slow ion temperature up to 100 keV. At the same time, the second $P = 0$ location moves to higher wave number such that a fourth growth rate peak appears at the wave number near $k_{\rho_0} = 9$. The growth rate peaks are shown in Fig. 5 as a function of the slow ion temperature. When the slow ion temperature increases to overcome the Alfvénic condition at 48.5 keV, the second growth rate peak appears first. The growth rate of this peak increases with the slow ion temperature and then decreases when the slow ion temperature is higher than 60 keV. So far, the numerical result agrees well with the prediction of the cubic instability. Then, the characteristic of this peak transits to the coupled quadratic instability when the slow ion tem-
perature reaches 75 keV. This transition is due to the coefficient \( P \) being too high for the condition of the cubic instability and the coefficient \( D \) of the slow ion resonance is high enough to satisfy the coupled quadratic instability requirement. It is also interesting to note that the fourth growth rate peak appears when the instability transition occurs at 75 keV. The first growth rate peak starts to appear when the slow ion temperature is higher than 55 keV. For the slow ion temperature up to 100 keV, this first growth rate peak remains as a cubic instability and has not transited to the coupled quadratic instability as in contrast to the 3.02 MeV proton case.

C. Slow ion density

Figure 6 shows the growth rate spectrums as a function of the wave number for different slow ion densities by keeping the ratio of the fast and slow ion densities as a constant. The result is similar to what happens for raising the slow ion temperature. However, there are still some differences. Near the threshold, the location of the second growth rate peak moves to slightly higher wave number as in contrast to that of increasing the slow ion temperature, because it is proportional to the square root of the slow ion density, not the temperature [Eq. (14)]. The movement of the location of the first growth rate peak is not as fast as the case of the slow ion temperature in terms of same slow ion pressure. Also, the wave number corresponding to the fourth growth rate peak moves higher and much faster than that of increasing the slow ion temperature. These results can be understood by the coefficient \( P \) as a function of the wave number for different slow ion densities. The coefficient \( P \) equals the frequency mismatch and the ratio of the coefficients \( D \) and \( A \). The coefficient \( D \) is proportional to the slow ion density while the coefficient \( A \) has one term proportional to the square of the wave number and the other term proportional to the slow ion density. At the low wave number regime, the second term in the coefficient \( A \) dominates so that the ratio of the coefficient \( D \) and \( A \) do not vary with an increase of the slow ion density, as the coefficient \( P \). However, at high wave number regime, the first term in the coefficient \( A \) dominates so that both the coefficient \( P \) and the ratio of the coefficients \( D \) and \( A \) increases with the slow ion density. Thus, the movement of the first \( P=0 \) point is slower toward lower wave number while that of the second \( P=0 \) point is faster toward higher wave number.

The peak growth rate as a function of the slow ion density is shown in Fig. 7. When the slow ion density is higher than the threshold of \( 4.45 \times 10^{13} \) cm\(^{-3} \), the second growth rate peak appears and increases with the slow ion density. It reaches its maximum at the slow ion density of \( 5.3 \times 10^{13} \) cm\(^{-3} \) and starts to decrease. Up to \( 5.6 \times 10^{13} \) cm\(^{-3} \), the numerical result agrees well with the prediction of the cubic instability. Then, the second growth rate peak starts to transit and becomes a coupled quadratic instability at \( 6.4 \times 10^{13} \) cm\(^{-3} \). When the slow ion density is higher than \( 6.2 \times 10^{13} \) cm\(^{-3} \), we find the fourth growth rate peak at high wave number. This fourth peak increases with the slow ion density at first and then decreases, and it is a cubic instability. When the slow ion density is higher than \( 5 \times 10^{13} \) cm\(^{-3} \), the first growth rate peak appears as a cubic instability and its value increases with the slow ion density.

D. Magnetic field

Figure 8 shows the growth rate spectrums for different magnetic fields, while Fig. 9 shows the growth rate peaks as a function of the magnetic field. The instability transition of the second growth rate peak from the cubic to the coupled quadratic, the appearance of the first and fourth growth rate peaks, and other related phenomena are observed. As in con-
IV. SUMMARY

With a weaker driving from the 14.7 MeV proton, the surprising dominance of the second peak in relativistic electromagnetic ion cyclotron instability is studied by analytical theory and numerical calculation. A homogeneous plasma is studied here to illustrate the interesting new physics against conventional wisdom while the impact of inhomogeneity is being addressed. Alfvénic condition requires the instability threshold of the slow ion temperature increases with the fast ion energy. Thus, an analytical theory has been developed to study the finite slow ion temperature effect that affects the resonant slow ion inertia, but not the nonresonant. This effect is larger at higher wave number so that the peak of the coefficient $P$ is moved to higher wave number (e.g., $kP_0=7$) near the second Bessel and growth rate peak instead of the first peak (e.g., $kP_0=4$). Because the Alfvénic condition resulted from the condition $P=0$ for the cubic instability and $P$ begins as a negative value is increased with the slow ion temperature, the Alfvénic condition is first satisfied at the regime near the peak of $P$. Therefore, the numerical calculation show that, while the first growth rate peak remains stable, the second peak overcomes the Alfvénic condition first and dominates the behavior near the threshold. This is surprising because the relativistic driving term for the second peak is much smaller than that for the first peak.

As the slow ion temperature is increased, the first growth rate peak appears, the second peak transits from the cubic instability to the coupled quadratic instability, and then the fourth growth rate peak appears. But, up to twice the slow ion temperature threshold, the first growth rate peak remains as a cubic instability and we do not find the third growth rate peak, as in contrast to the 3.02 MeV proton case. Similar results are observed by increasing the slow ion density or by decreasing the magnetic field. When the slow ion temperature is increased further, the first (fourth) peak moves to a lower (higher) wave number faster (slower) than the increase of the fast ion density or the decrease of the magnetic field. The required percentage change of the magnetic field is smaller than that of the slow ion density and temperature. Again, the first peak remains as a cubic instability.

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relativistic gyrobroadening remains similar when the magnetic field is nonuniform.

