Spectrum analysis for high-order cladding modes based on long-period fiber gratings

Jen-Fa Huang
Yue-Jing He
National Cheng Kung University
Department of Electrical Engineering
Tainan, Taiwan

Yu-Lung Lo, MEMBER SPIE
National Cheng Kung University
Department of Mechanical Engineering
Tainan, Taiwan
E-mail: loyl@mail.ncku.edu.tw

Abstract. The spectrum characteristic for the coupling between the core mode HE_{11} and high-order cladding modes based on long-period fiber gratings (LPGs) is investigated with the aim of supplying a concrete concept to design narrow-bandwidth optical devices. In contrast to the better-known coupling between the core mode and low-order cladding modes, numerical simulation has shown that a 0.4-nm FWHM can be achieved by properly designing the period of the LPG in choosing the high-order cladding modes. In any case, in order to ensure that the analytic result for high-order cladding modes is meaningful, the validity of two-mode coupled-mode equations is also proved from the three-mode coupled-mode equations. According to this concept, LPG can be used to design not only gain flatteners that have wide bandwidth but also optical communication components that have narrow bandwidth to meet the ITU standard for a DWDM system. © 2006 Society of Photo-Optical Instrumentation Engineers. [DOI: 10.1117/1.2354499]

Subject terms: long-period fiber grating (LPG); dense wavelength-division multiplexing.

Paper 050959R received Dec. 7, 2005; revised manuscript received Feb. 5, 2006; accepted for publication Feb. 8, 2006; published online Oct. 5, 2006.

1 Introduction

It is well known that a long-period fiber grating (LPG) written periodically by ultraviolet light into the core of an optical fiber can couple the power among the copropagating modes. The coupling between the core mode HE_{11} and cladding modes has been used extensively for band rejection filters, gain flatteners, and dispersion compensators.\textsuperscript{1–3}

For a given azimuthal number \( l \), there are typically several hundred cladding modes at near-infrared wavelength in a traditional fiber. According to the fundamental property of modes, the higher the order of the cladding mode, the more the field can extend. Obviously, a high-order cladding mode is more sensitive to variation of the environment than a low-order mode. Therefore, a variety of sensors based on high-order cladding modes have been proposed.\textsuperscript{4–7}

So far, all investigations of the spectra of LPGs have emphasized the coupling between the core mode HE_{11} and low-order cladding modes.\textsuperscript{8–11} It is well known that the LPG has considerably greater bandwidth than the reflection fiber Bragg grating, whose spectrum is formed by coupling the forward-propagating core mode to the backward-propagating core-cladding modes. Consequently, it seems impossible to use an LPG to design wavelength-division multiplexing (WDM) components, not to mention conforming to the dense wavelength-division multiplexing (DWDM) standard.

According to the fundamental property of modes, a high-order cladding mode is more sensitive to the variation of wavelength than a low-order one.\textsuperscript{11,12} On the other hand, a high-order cladding mode can depart from the phase-matching condition more rapidly than a low-order mode when the wavelength changes. This simple fact implies that it is possible for an LPG to possess as narrow a bandwidth as an FBG only if a high-order cladding mode is used. In this paper, we use accurate mathematical theory to investigate and quantify the spectral characteristics of high-order cladding modes with the aim of supplying a concrete concept to design narrow-bandwidth optical devices.

The remainder of this paper is constructed as follows. In Sec. 2 we utilize the dispersion relation and the coupling coefficient to describe the fundamental characteristics of the cladding modes. According to the general coupled-mode equation, we know that the occurrence of coupling between two copropagating modes must simultaneously meet two essential conditions, namely, the phase-matching condition and a proper cross coupling constant. Obviously, the dispersion relation and the coupling coefficient of a cladding mode play rather important roles in determining the transmission bandwidth. In Sec. 3 we concretely quantify the bandwidth characteristics of low-order and high-order cladding modes on condition that two-mode coupling equations are valid. In any case, in order to ensure that the analytic result for high-order cladding modes is meaningful, the assumption that the two-mode coupling equations are valid is also proved from the three-mode coupled-mode equations. Finally, in Sec. 4 we discuss the results and propose a concept to design a communication component with narrow bandwidth and small dimensions.

2 Fundamental Characteristics and Coupling Coefficients

In this section, we introduce in detail two basic and important properties of cladding modes. A three-layer cylindrical optical fiber is described by the following parameters: \( n_1 = 1.4602, \; n_2 = 1.4443, \; n_3 = 1.00, \; a_1 = 2.25 \, \mu m, \; a_2 \).
A dielectric perturbation can be written as

\[ \frac{dA_\mu}{dz} = i \sum_\nu A_{\nu,\mu} \exp[i(\beta_\nu - \beta_\mu)z] \]  

(1)

with

\[ K_{\nu,\mu} = \frac{w \Delta e}{4} \int_0^{2\pi/a} \int_0^{a_1} r(E_\nu E_\mu^* + E_\nu^* E_\mu) \, dr \, d\phi , \]

(2)

where \( A_\mu(z) \) is the amplitude for the transverse mode field, and \( K_{\nu,\mu} \) is the coupling coefficient between modes \( \nu \) and \( \mu \). In this paper, we begin the bandwidth analysis by assuming that only two modes (one core mode and one cladding mode) are involved at each wavelength. Therefore, the two-mode coupled-mode equations can be obtained by simplifying the general coupled-mode equations and written as follows:

\[ \frac{dA_{cc}(z)}{dz} = i \sigma_{cc} A_{cc}(z) + ik_{cc} A_{cc}(z) \]

\[ \times \exp \left[ -i \left( \beta_{cc} - \beta_0 - \frac{2\pi}{\Lambda} \right) \right] , \]

(3)

\[ \frac{dA_{c1}(z)}{dz} = ik_{c1} A_{c1}(z) \exp \left[ i \left( \beta_{c1} - \beta_0 - \frac{2\pi}{\Lambda} \right) \right] + i \sigma_{c1} A_{c1}(z) \]

with

\[ \sigma_{cc} = \frac{w \rho \mu_0^2 \sigma}{2} \int_0^{2\pi/a} \int_0^{a_1} r(E_r E_r^* + E_\phi E_\phi^*) \, dr \, d\phi , \]

(4)

\[ \sigma_{c1} = \frac{w \rho \mu_0^2 \sigma}{2} \int_0^{2\pi/a} \int_0^{a_1} r(E_r E_{r,1}^* + E_\phi E_{\phi,1}^*) \, dr \, d\phi , \]

(5)

\[ k_{c1} = \frac{w \rho \mu_0^2 \sigma}{4} \int_0^{2\pi/a} \int_0^{a_1} r(E_r E_{r,1} + E_\phi E_{\phi,1}) \, dr \, d\phi , \]

(6)

where \( A_{cc}(z) \) is the amplitude of the core mode HE_{11}, \( A_{c1}(z) \) is the amplitude of the cladding mode \( \nu \), \( \sigma_{cc} \) is the dc coupling coefficient of HE_{11}, \( k_{c1} \) is the ac coupling coefficient between HE_{11} and cladding mode \( \nu \), \( \sigma_{c1} \) is the dc coupling coefficient of cladding mode \( \nu \), \( \sigma \) is the UV-induced refractive index variation, and \( \Lambda \) is the period of the long-period fiber grating.

The dispersions of the effective indices of cladding modes are shown in Fig. 1(a) for low-order modes \( \nu = 1, \ldots, 10 \), and in Fig. 1(b) for high-order modes \( \nu = 103, \ldots, 111 \). From Fig. 1, we can find that the high-order cladding modes are much more sensitive to wavelength than the low-order ones. If we further combine the phase-matching condition with this phenomenon, it also provides us with an intuitive idea that high-order cladding mode may possess narrow bandwidth.

When a uniform long-period fiber grating is induced in the fiber core, the general coupled-mode equation that describes the changes in the copropagating amplitudes of a mode \( \mu \) resulting from the presence of other modes \( \nu \) near a dielectric perturbation can be written as

\[ \frac{dA_\mu}{dz} = i \sum_\nu A_{\nu,\mu} \exp[i(\beta_\nu - \beta_\mu)z] \]  

(1)

with

\[ K_{\nu,\mu} = \frac{w \Delta e}{4} \int_0^{2\pi/a} \int_0^{a_1} r(E_\nu E_\mu^* + E_\nu^* E_\mu) \, dr \, d\phi , \]

(2)

where \( A_\mu(z) \) is the amplitude for the transverse mode field, and \( K_{\nu,\mu} \) is the coupling coefficient between modes \( \nu \) and \( \mu \). In this paper, we begin the bandwidth analysis by assuming that only two modes (one core mode and one cladding mode) are involved at each wavelength. Therefore, the two-mode coupled-mode equations can be obtained by simplifying the general coupled-mode equations and written as follows:

\[ \frac{dA_{cc}(z)}{dz} = i \sigma_{cc} A_{cc}(z) + ik_{cc} A_{cc}(z) \]

\[ \times \exp \left[ -i \left( \beta_{cc} - \beta_0 - \frac{2\pi}{\Lambda} \right) \right] , \]

(3)

\[ \frac{dA_{c1}(z)}{dz} = ik_{c1} A_{c1}(z) \exp \left[ i \left( \beta_{c1} - \beta_0 - \frac{2\pi}{\Lambda} \right) \right] + i \sigma_{c1} A_{c1}(z) \]

with

\[ \sigma_{cc} = \frac{w \rho \mu_0^2 \sigma}{2} \int_0^{2\pi/a} \int_0^{a_1} r(E_r E_r^* + E_\phi E_\phi^*) \, dr \, d\phi , \]

(4)

\[ \sigma_{c1} = \frac{w \rho \mu_0^2 \sigma}{2} \int_0^{2\pi/a} \int_0^{a_1} r(E_r E_{r,1}^* + E_\phi E_{\phi,1}^*) \, dr \, d\phi , \]

(5)

\[ k_{c1} = \frac{w \rho \mu_0^2 \sigma}{4} \int_0^{2\pi/a} \int_0^{a_1} r(E_r E_{r,1} + E_\phi E_{\phi,1}) \, dr \, d\phi , \]

(6)

where \( A_{cc}(z) \) is the amplitude of the core mode HE_{11}, \( A_{c1}(z) \) is the amplitude of the cladding mode \( \nu \), \( \sigma_{cc} \) is the dc coupling coefficient of HE_{11}, \( k_{c1} \) is the ac coupling coefficient between HE_{11} and cladding mode \( \nu \), \( \sigma_{c1} \) is the dc coupling coefficient of cladding mode \( \nu \), \( \sigma \) is the UV-induced refractive index variation, and \( \Lambda \) is the period of the long-period fiber grating.

Note that throughout this paper the long-period fiber grating is assumed to be a circularly symmetric index perturbation, so that the coupling interaction only occurs between HE_{11} and the cladding modes with azimuthal order \( l = 1 \). Figure 2(a) and 2(b) show the ac coupling coefficient between the core mode HE_{11} and cladding mode \( \nu \), and the dc coupling coefficient for cladding mode \( \nu \), respectively, at the wavelength \( \lambda = 1550 \) nm. From Fig. 2, we can clearly
between cladding mode and first-zeros bandwidth? When we change the period of the LPG to choose a different high-order cladding mode, is the bandwidth of the LPG getting narrower or wider? In order to interpret in detail how the dispersion of the effective indices of cladding modes and the ac coupling coefficient $k_{cl-cl}$ influence the bandwidth of the LPG, we combine coupled-mode theory and these two parameters. With the use of the boundary conditions for the LPG, $A_{co}(z=0) = 1$ and $A_{cl}(z=0) = 0$, the bar power transmission can be written as follows:

$$t_{cl}(z) = \left| A_{co}(z) \right|^2 / \left| A_{co}(0) \right|^2$$

$$= \cos^2 \left( (k_{cl-co}^v + \hat{\sigma}^2)^{1/2} z \right) + \frac{\hat{\sigma}^2}{k_{cl-co}^v + \hat{\sigma}^2} \sin^2$$

$$\times \left[ (k_{cl-co}^v + \hat{\sigma}^2)^{1/2} z \right],$$

with

$$\hat{\sigma} = \frac{\pi (n_{co} - n_{cl}^v)}{\lambda} + \frac{\sigma_{co-co} - \sigma_{cl-cl}^v}{2} - \frac{\pi}{\Lambda},$$

where $\hat{\sigma}$ is the general dc self-coupling coefficient, $n_{co}$ is the effective refractive index of the core mode HE$_{11}$, $n_{cl}^v$ is the effective refractive index of cladding mode $v$.

As stated in the previous section, the variation in the basic characteristics, $n_{co}^v(\lambda)$ and $\sigma_{cl-cl}^v(\lambda)$, for high-order cladding modes is larger than that for low-order modes when the wavelength changes. Therefore, $\hat{\sigma}$ should no longer be regarded as constant. Figure 3(a) shows the general dc self-coupling coefficients for low-order modes $v = 1, \ldots, 10$ when the period $\Lambda$ of the LPG is designed to couple the HE$_{11}$ mode to the $v = 7$ cladding mode at 1550 nm, and Fig. 3(b) shows them for high-order modes $v = 103, \ldots, 111$ when $\Lambda$ of LPG is designed to couple the HE$_{11}$ mode to the $v = 107$ cladding mode at 1550 nm.

In Fig. 3(a), we find that the general dc self-coupling coefficient decreases with increasing wavelength. This arises mainly because the decrease in the effective index of a core mode when the wavelength increases is larger than that in the effective index of a low-order cladding mode. In contrast, the decrease in the effective index of a core mode when the wavelength increases is smaller than that in the effective index of a high-order cladding mode, so the general dc self-coupling coefficient in Fig. 3(b) increases with increasing wavelength.

From Eq. (7), the minimum bar power transmission occurs when $\hat{\sigma} = 0$. Figure 4(a) shows the position of the minimum transmission spectrum for low-order cladding modes $v = 1, 3, 5, 7, 9$ when the period $\Lambda$ of the LPG is designed to couple the HE$_{11}$ mode to the $v = 7$ cladding mode at 1550 nm. Moreover, Fig. 4(b) shows the position of the minimum transmission spectrum for high-order cladding modes $v = 103, \ldots, 111$ when $\Lambda$ is designed to couple the HE$_{11}$ mode to the $v = 107$ cladding mode at 1550 nm. Most notably, among the low-order cladding modes, only the odd-order cladding modes are considered, since the coupling from HE$_{11}$ to the low-order even modes is rather weak. From Fig. 4, we clearly find that the sequence of positions for the high-order cladding modes is in contrast to that for the low-order modes.

### 3 Bandwidth Characteristics of Cladding Modes

According to the preceding analysis, the dispersion of the effective indices of cladding modes and the ac coupling coefficient $k_{cl-co}$ are two major keys in investigating narrow bandwidth. Physically, they provide us with an intuitive and reasonable picture. What, however, is the direct relation
From Eq. (7), the bandwidth of the resonance is given by twice the change in wavelength that causes the general dc self-coupling coefficient \( \hat{\sigma} \) to vary from \( \hat{\sigma} = 0 \) to a value of either \( \pm \left[ \frac{\pi}{L} \left( k_{\text{cl-co}} \right)^2 \right]^{1/2} \) (for the bandwidth between the first zeros on either side of the resonance). In addition, in order to couple a core mode to the cladding mode that we desire, the period and length of the long-period fiber grating should be properly designed with

\[
\Lambda = \frac{2\pi}{\sigma_{\text{co-co}} - \sigma_{\text{cl-cl}}^{\nu} + (2\pi/\lambda)(n_{\text{co}} - n_{\text{cl}})^{\nu}},
\]

(9)

\[
L = \frac{\pi}{2k_{\text{cl-co}}^{\nu}}.
\]

(10)

So as to quantify specifically the direct relation between cladding mode and first-zeros bandwidth at 1550 nm, Fig. 5(a) shows the first-zeros bandwidth for low-order cladding modes \( \nu = 5, 7, 9 \) when the period \( \Lambda \) of the LPG is designed to couple the HE_{11} mode to the \( \nu = 5, 7, 9 \) cladding modes, respectively, and Fig. 5(b) shows the first-zeros bandwidth for high-order cladding modes \( \nu = 106, 107, 108 \) when \( \Lambda \) is designed to couple the HE_{11} mode to the \( \nu = 106, 107, 108 \) cladding modes, respectively. In Fig. 5, we clearly find that the high-order cladding mode can possess narrower bandwidth than the low-order cladding mode, mainly because the variation in the general dc self-coupling coefficient of a high-order cladding mode is much larger than that of a low-order cladding mode.

In this paper, we propose a concept to design communication components with narrow bandwidth and small dimensions. First, in order to minimize the size of components, we need to choose a proper \( k_{\text{cl-co}}^{\nu} \) according to Eq. (10). Second, in order to minimize their bandwidth, we need to choose the higher-order cladding mode that yields the same \( k_{\text{cl-co}}^{\nu} \) as a first step, by properly designing the period of the LPG. For completeness, we plot the transmission spectrum of low-order cladding modes in Fig. 6(a) with the same parameters as in Fig. 5(a), and the transmission spectrum of high-order cladding modes in Fig. 6(b) with same parameters as in Fig. 5(b). From the result of numerical simulation, a 0.4-nm FWHM can be obtained by properly designing the period of the LPG through choice of the high-order cladding.

As stated earlier, the bandwidth analysis throughout this section is based on the assumption that the two-mode coupled-mode equation is valid. The assumption that only
two modes are involved at each wavelength is mainly aimed to simplify the analysis of the bandwidth of the LPG. After we use the two-mode coupled-mode equations for that purpose, it is necessary to check whether this assumption is valid. If not, all results derived from it will be pointless. In order to confirm it, we plot the transmission spectrum of low-order cladding modes \( \nu = 1, 3, 5, 7, 9 \) in Fig. 7(a) when the period of the LPG is designed to couple the \( \text{LP}_{01} \) mode to cladding modes \( \nu = 5, 7, 9 \), respectively, and (b) for high-order cladding modes \( \nu = 106, 107, 108 \), when the period of LPG is designed to couple the \( \text{LP}_{01} \) mode to cladding modes \( \nu = 106, 107, 108 \), respectively.

Fig. 5 First-zeros bandwidth at 1550 nm: (a) for low-order cladding modes \( \nu = 5, 7, 9 \) when the period of the LPG is designed to couple the \( \text{LP}_{01} \) mode to cladding modes \( \nu = 5, 7, 9 \), respectively, and (b) for high-order cladding modes \( \nu = 106, 107, 108 \), when the period of LPG is designed to couple the \( \text{LP}_{01} \) mode to cladding modes \( \nu = 106, 107, 108 \), respectively.

In order to clarify the process that we use to analyze the bandwidth of the LPG in this paper, a flow diagram is shown in Fig. 8. In this diagram, the red path and black path (color online only) express the processes of analyzing bandwidth for high-order and for low-order cladding modes, respectively. From the red path, we can find that event B for high-order cladding is not in contradiction with event A. But can we say with fair certainty that it is valid for high-order cladding modes to use two-mode coupled-mode equations to analyze the bandwidth of the LPG? Because it is just an assumption that only two cladding modes meet the phase-matching conditions at each wavelength, we may suspect that more than two do so. Consequently, we further use the three-mode coupled-mode equations (assuming that only three modes—one core mode and two cladding modes—are involved at each wavelength) to check the two-mode coupling equations. If the three-mode coupled-mode equations and the two-mode coupled-mode equations are identical in bandwidth spectrum, we can say that it is correct and sufficient to use the latter to analyze the bandwidth of the LPG. On the contrary, if three-mode coupled-mode equations and two-mode coupled-mode equations differ in bandwidth spectrum, it means that two-mode coupled-mode equations used to analyze the bandwidth of LPG are not enough, and we need three-mode coupled-mode equations. Then we must ask: are three-
mode coupled-mode equations enough? We test that by examining the four-mode coupled-mode equations again. According to what has been stated, the conclusion is that we can use $n+1$-mode coupled-mode equations to check whether the $n$-mode coupled-mode equations are enough for the bandwidth. The notation $A \rightarrow B$ denotes that event $A$ implies event $B$, and it is equivalent to $-B \rightarrow -A$.

$$\frac{dA_{co}}{dz} = iK_{co-co}A_{co} + iK_{cl-co}^{107} A_{cl}^{107} \exp[i(\beta_{cl}^{107} - \beta_{co})z]$$
$$+ iK_{cl-co}^{108} A_{cl}^{108} \exp[i(\beta_{cl}^{108} - \beta_{co})z],$$

$$\frac{dA_{cl}^{107}}{dz} = iK_{co-cl}^{107} A_{co} \exp[i(\beta_{co} - \beta_{cl}^{107})z] + iK_{cl-cl}^{107} A_{cl}^{107}$$
$$+ iK_{cl-cl}^{108-107} A_{cl}^{108} \exp[i(\beta_{cl}^{107} - \beta_{cl}^{108})z] + iK_{cl-cl}^{108} A_{cl}^{108}.$$  

It is shown clearly in Fig. 9 that the transmission spectrum of the two-mode coupled-mode equations is coincident with that of the three-mode coupled-mode equations.
Huang, He, and Lo: Spectrum analysis for high-order cladding modes…

Fig. 9 Numerical results of two-mode and three-mode coupled-mode equations for v = 107 and v = 108 cladding modes.

4 Conclusions

In contrast to the coupling between core mode and low-order cladding modes, the spectrum characteristic of high-order cladding mode has been analyzed in detail by using rigorous mathematical theory. In order to ensure that the analytic result for high-order cladding modes is meaningful, the validity of the two-mode coupled-mode equations is also proved by use of the three-mode coupled-mode equations.

Finally, we propose two steps to design an LPG that possesses narrow bandwidth. In the first step, in order to minimize the size of the LPG, we need to choose proper $k_{cl-co}$ because the smaller the $k_{cl-co}$ the longer the LPG that can completely couple the HE_{11} mode to the cladding mode. In second step, in order to minimize the bandwidth of LPG, we need to choose the higher-order cladding mode that yields the same $k_{cl-co}$ as in the first step, by choosing the period of the LPG. Numerical simulation shows that a 0.4-nm FWHM can be obtained by properly designing the period of the LPG through choice of the high-order cladding.

An LPG can be used to design not only gain flatteners that have wide bandwidth, but also optical communication components that have narrow bandwidth to meet the ITU standard.

Acknowledgments

The authors gratefully acknowledge the support provided for this study by the Ministry of Education’s Program for Promoting Academic Excellence of Universities under grant No. A-91-E-FA08-1-4, and by the National Science Council under grant No. NSC94-2215-E-006-049. Also, funding from the Advanced Optoelectronic Technology Center, National Cheng Kung University under projects from the Ministry of Education and the National Science Council (NSC 95–219–M-009–008) of Taiwan is gratefully acknowledged.

References


Jen-Fa Huang received his MASc and PhD degrees from the Department of Electrical Engineering at the University of Ottawa, ON, Canada, in 1981 and 1985, respectively. Since 1991, he has been with the Department of Electrical Engineering at the National Cheng Kung University, Taiwan, where he is currently a conjoint professor of the Institute of Computer and Communication Engineering and the Institute of Opto-Electronic Science and Engineering. Previous to 1991, he was with MPB Technologies, Montreal, PQ, Canada, in the Optical Communication Laboratories, working on the TAT-9 transatlantic undersea lightwave transmission project. His research interests are mainly in the areas of optical communications, all-optical data networking, and fiber-optic sensors.

Yue-Jing He received his MS degree in the Department of Communication Engineering at National Chiao-Tung Taiwan University, Taiwan, in 2000. He is currently working towards his PhD degree in the area of fiber-optic networking communications. His major interests are in DWDM networking devices, fiber Bragg grating filters, long-period fiber grating sensors, and multiuser communication systems.

Yu-Lung Lo received his BS degree from National Cheng Kung University, Tainan, Taiwan, in 1985, and the MS and PhD degrees in mechanical engineering from the Smart Materials and Structures Research Center, University of Maryland, College Park, in 1992 and 1995, respectively. After graduation, he joined the Industrial Technology Research Institute (ITRI), Opto-Electronics and Systems Laboratories, working on fiber optic smart structures. He has been a member of the faculty of the Mechanical Engineering Department, National Cheng Kung University, since 1996. His research interests are in the areas of experimental mechanics, fiber-optic sensors, smart structures, optical techniques in precision measurements, electronic packaging, and MEMS. He has authored more than 50 technical publications and filed five patent disclosures. He is a member of SPIE and SEM.