A Smooth Transition Autoregressive Conditional Duration Model

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Abstract

This study presents a novel model for analyzing duration data, called the smooth transition autoregressive conditional duration model of price and duration, which considers past price changes and durations. The model enables the process of the conditional expected duration to switch in a smooth transition way, broadening the autoregressive conditional duration (ACD) model in Engle and Russell (1998). The model is applied to empirical data, and estimation results indicate that the process of the expected duration is nonlinear. The expected trade duration behavior on the market opening is affected by past trade durations, while the expected trade duration behavior during the trading hours is affected by past price changes and trade durations. Expected trade durations are much more persistent in the upward market compared to the downward market. Shocks to trade durations are more persistent on the market opening and gradually decrease in the downward market.

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1 Introduction

Asset price information is revealed to the market via investor trades. Thus, retrieving and extracting information from the order sequence can help understand the assimilation of asset price information. Easley and O’Hara (1992) established a model demonstrating how the time between trades contains information owing to event uncertainty. Accordingly, the correlation between the time between trades and information occurs when the order sequence is connected to the time between trades.

The seminal paper by Engle and Russell (1998) established the autoregressive conditional duration model (ACD) for capturing the dynamic behavior of trade durations (time between trades). The ACD model considers trade intensity and expresses the conditional expectation of trade durations as an autoregressive relationship of past trade durations. Such modeling for high-frequency data has proven useful in studying the empirical implications of market microstructure theory. Engle and Russell (1997) applied the ACD model to investigate foreign exchange quotes and support the asymmetric information model of price setting. Moreover, Hamelink (1998) found a significant correlation between the durations and returns of French CAC 40 using the ACD model. Consequently, durations provide a channel for asset price information.

Following Engle and Russell (1998), several duration models have been developed. Jasiak (1998) considered the long range of time dependence in trade durations and introduced a fractionally integrated ACD model. Bauwens and Giot (2000) considered the logarithmic ACD model, which is able to avoid the positivity constraint in parameters. Furthermore, Grammig and Maurer (2000) applied the Burr distribution to increase the flexibility of the conditional hazard function. Meanwhile, Zhang et al. (2001) proposed a threshold ACD (TACD) model in which the nonlinearity of the ACD model depends on past trade durations. Furthermore, Bauwens and Giot (2003) proposed an asymmetric ACD model in which the asymmetry depends upon price change states. Bauwens and Veredas (2004) introduced a stochastic conditional duration (SCD) model involving a mixture of distributions, which allows the conditional expected duration to be random. Ghysels et al. (2004) also developed an ACD model with a
mixture of distributions, termed the stochastic volatility duration (SVD) model, in which the conditional mean and overdispersion are driven by two dynamic factors. Considering the relationship between price movements and trade durations, Russell and Engle (2005) developed a model combining the autoregressive conditional multinomial model (ACM) and the autoregressive conditional duration (ACD) model, called ACM-ACD, in which discrete price movements are established as a multinomial model. Generally, the variants of the above ACD model depend on the features of nonlinearity, past price changes, past durations, or distribution assumptions. However, none of these variants integrate features of nonlinearity, past price changes, and past durations.

Previous empirical studies have indicated that past price changes and past trade durations are important influences on the trade duration process. Hamelink (1998) studied the French CAC 40 by clustering duration regimes based on returns and durations. He found that the duration processes differ among regimes. Zhang et al. (2001) applied the threshold ACD (TACD) model to the stock of IBM using the Trades, Orders, Reports, and Quotes (TORQ) data set and found that nonlinearity in the ACD model identified by past trade durations delineates the trade duration process much better than the ACD model of Engle and Russell (1998). Meanwhile, Bauwens and Giot (2003) applied the asymmetric ACD model to the stocks of IBM and Disney and found asymmetries in the trade duration process during the upward and downward price changes. Examining Airgas stock (ticker symbol ARG) using the ACM-ACD model, Russell and Engle (2005) found that past price returns influence trade durations. Finally, Chiang et al. (2006) studied futures markets trade durations using the ACD model and found that the trade duration dynamics vary with the size of price changes. Consequently, the trade duration process is not a simple linear function of past trade duration, but rather can be affected by the sizes of past price changes and past trade durations.

This study proposes a nonlinear type of autoregressive conditional duration model, termed the smooth transition autoregressive conditional duration model of price and duration (STACD-PD), which considers the previous price change and duration. The nonlinearity in the STACD-PD model enables the process of the conditional expected trade duration to follow a regime-switching behavior. Meanwhile, the regime switches in a smooth transition process in which the
smooth transition probabilities vary according to trends in price changes and durations. Originated by Bacon and Watts (1971) and popularized and in-depth by Teräsvirta (1994) and Granger and Teräsvirta (1993), the smooth transition model characterizes the regime-switching process in terms of gradualness and continuity rather than instancy as in the Markov switching model or discontinuity as in the threshold model. The gradualness and continuity properties increase the suitability of the smooth transition model for high-frequency data because intraday trades are highly intensive and likely to be persistent\textsuperscript{1} (Lin et al., 1995, Hasbrouck, 1991, Choi et al., 1988). Consequently, the STACD-PD model is able to capture the process of the expected trade duration subject to variations in price and duration.

The STACD-PD model is employed to investigate intraday transaction data for the stock of IBM, traded on the New York Stock Exchange (NYSE). Empirical results demonstrate that the process of expected trade durations is nonlinear. The behavior of expected trade durations differs between the market opening and the trading hours. On the market opening, the expected trade duration behavior is affected by past trade durations but not past price changes. Additionally, the expected trade durations are more persistent with increased past trade durations. During the trading hours, the expected trade duration behavior is affected by past price changes and past trade durations. Expected trade durations are more persistent in the upward market than in the downward market. Furthermore, shorter past trade durations and downward price changes will reduce trading frequency, which may be caused by the short sale constraints and more informed trading. The data are grouped into distinct regimes based on past price changes and trade durations. The inventory and order processing costs are higher in the longer duration regime on the market opening while information asymmetry is more severe in the downward market during the trading hours. Shocks to trade durations are more persistent on the market opening and weaken gradually in the downward market.

The remainder of this paper is organized as follows. Section 2 reviews the ACD model and the logarithmic ACD model. This section also constructs the specification and likelihood functions of the STACD-PD model. Section 3 then

\textsuperscript{1} The order persistence refers to the order submission phenomenon of buy (sell) orders tending to follow buy (sell) orders.
develops the specification tests of the STACD-PD model. Next, Section 4 presents and discusses the empirical results, and Section 5 gives conclusions.

2 Specifications of the STACD-PD Model

The autoregressive conditional duration (ACD) model of Engle and Russell (1998) is an intraday duration model that accommodates the clustering property of durations. Fundamentally, Engle and Russell (1998) consider a sequence of arrival times and assume that the realized duration can be expressed as a random process in which the conditional expected duration is an autoregressive process of past conditional and actual durations. Let \( x_i = t_i - t_{i-1} \) represent the duration between two successive arrival times, \( t_i \) and \( t_{i-1} \). The observed trade duration in the ACD model can then be expressed as follows:

\[
x_i = \Theta_j \varepsilon_j,
\]

where \( \{\varepsilon_j\} \) are positive i.i.d. random variables with a certain distribution over \((0, \infty)\), and \( E(\varepsilon_j) = \mu \). Therefore, the conditional expectation of \( x_i \), given a certain distribution of \( \{\varepsilon_j\} \) and an information set of \( \Omega_{i-1} \) up to the transaction \( i-1 \), \( E(x_i | \Omega_{i-1}) \), is \( \psi_i \) which equals \( \Theta_j \mu \) and is specified as follows:

\[
\psi_i = w + \sum_{j=1}^{n} \alpha_j x_{i-j} + \sum_{k=1}^{a} \beta_k \psi_{i-k},
\]

where the root of the difference equation lies outside the unit circle and \( w \) is larger than zero. Those two conditions guarantee that the durations have a positive unconditional mean. Meanwhile, Bauwens and Giot (2000) propose a logarithmic ACD model, known as log-ACD, in which the positivity constraint in the ACD model can be relaxed. The observed duration in the log-ACD model is expressed as follows:

\[
x_i = \Theta_j \varepsilon_i = \exp(\phi) \varepsilon_i,
\]

where the logarithm of the conditional mean of \( x_i \) given a certain distribution of \( \{\varepsilon_j\} \) and an information set of \( \Omega_{i-1} \) up to the transaction \( i-1 \), \( \ln[E(x_i | \Omega_{i-1})] \),
is \( \psi_i \), which equals \( \phi + \ln u \). Additionally, the logarithm of the conditional mean, \( \psi_i \), is as follows:

\[
\psi_i = w + \sum_{j=1}^{q} \alpha_j \varepsilon_{i-j} + \sum_{k=1}^{k} \beta_k \psi_{i-k}.
\]  \hspace{1cm} (2.4)

Bauwens et al. (2003) presented existence conditions of the Log-ACD model. Particularly, \( |\beta| \) should be less than 1 to ensure the stationarity of \( \psi_i \) when \( p=q=1 \). Actually, the specification of the log-ACD model approximates the exponential GARCH setting in Nelson (1991), and assures the non-negative conditional expectation of the trade durations.

This study considers a new nonlinear ACD model that includes the feature of log-ACD and uses information contained in the price and duration movements. The observed trade duration in this new model adopts the form of the log-ACD model, as in Eqn. (2.3). Additionally, the logarithm of the conditional expected trade duration is modeled as a smooth transition model. Consider a transaction \( i \) with a price change \( \delta_i = p_i - p_{i-1} \), where \( p_i \) and \( p_{i-1} \) are two successive transaction prices, and with a trade duration \( x_i = t_i - t_{i-1} \), where \( t_i \) and \( t_{i-1} \) are two successive arrival times. The unexpected trade durations, \( \varepsilon_i = x_i / \Theta_i \), are assumed to be \( i.i.d \). with some distribution over \((0, \infty)\) as in Engle and Russell (1998). The conditional duration that is affected by the price change can then be modeled as the following specification using a smooth transition probability \( G_i(\delta_{i-1} \omega; \gamma_i, c_i) \): \( i \)

\[
\psi_{i(0)}^\delta = (\Psi_1(1 - G_i(\delta_{i-1} \omega; \gamma_i, c_i)) + \Psi_2 G_i(\delta_{i-1} \omega; \gamma_i, c_i))
\]  \hspace{1cm} (2.5)

\[
\Psi_1 = w_1 + \sum_{j=1}^{p} \alpha_j \varepsilon_{i-j} + \sum_{k=1}^{q} \beta_k \psi_{i-k} ,
\]

\( \Psi_2 \) The following standardizes the mean of \( \varepsilon_i \) to equal one. Consequently, \( \phi \) will equal \( \psi_i \).

\( \Psi_3 \) Bauwens et al. (2003) demonstrated that the absolute value of the maximum eigenvalue of the matrix involving coefficients of past durations should be less than one for the existence of moments of the Log-ACD model.

\( \Psi_4 \) For illustrative purposes, the price change is specified first and then the duration change effect is embedded. However, in actual practice the reverse procedure can be used.
$\Psi_2 = w_2 + \sum_{j=1}^{q} a_{2j} \psi_{i-j} + \sum_{k=1}^{q} \beta_{2k} \psi_{i-k}$,

where $G_1(\delta_{t-d}; \gamma_1, c_1)$ is a transition probability function with a state variable of price change, $\delta_{t-d}$, with $d1$ as a delay parameter of a positive integer, which is assumed to be at least twice differentiable and to range from 0 to 1; $\gamma_1$ is the smoothness parameter; and $c_1$ is a threshold value. The specification in Eqn. (2.5) models the expected duration process as a regime-switching process in which the expected duration process is driven by the price changes. This kind of specification incorporates the feature of allowing the conditional duration process to alternate with changing price. The most popular transition function is the following logistic function:

$$G_1(\delta_{t-d}; \gamma_1, c_1) = \frac{1}{1 + \exp(-\gamma_1(\delta_{t-d} - c_1))}$$

(2.6)

If $\gamma_1 \to 0$, the transition probability becomes 0.5, and then Eqn. (2.5) becomes linear. On the other hand, if $\gamma_1 \to \infty$, the transition probability function becomes a Heaviside function with a value of 1 when $\delta_{t-d} > c_1$ or a value of 0 when $\delta_{t-d} < c_1$. Besides the price change effect, the expected duration process may be affected by the duration itself, causing changes in the expected duration process. Therefore, Eqn. (2.5) is expanded as follows:

$$\psi_{d(t)} = \psi_{d(t)}^\delta (1 - G_2(x_{t-d2}; \gamma_2, c_2)) + \psi_{d2}^\delta G_2(x_{t-d2}; \gamma_2, c_2)$$

(2.7)

$$\psi_{d(t)}^\delta = \psi_{d(t)}^\delta (1 - G_1(\delta_{t-d}; \gamma_1, c_1)) + \psi_{d1}^\delta G_1(\delta_{t-d}; \gamma_1, c_1)$$

$$\psi_{d2}^\delta = \psi_{d2}^\delta (1 - G_1(\delta_{t-d}; \gamma_1, c_1)) + \psi_{d1}^\delta G_1(\delta_{t-d}; \gamma_1, c_1)$$

where $G_2(x_{t-d2}; \gamma_2, c_2)$ is a transition probability function with a state variable of the lagged duration, $x_{t-d2}$ with $d2$ as a delay parameter of a positive integer, $\gamma_2$ is the smoothness parameter, and $c_2$ is the threshold value associated with $G_2$. Therefore, $G_2$ can be written as follows:

$$G_2(x_{t-d2}; \gamma_2, c_2) = \frac{1}{1 + \exp(-\gamma_2(x_{t-d2} - c_2))}$$

(2.8)

Like $G_1$, $G_2$ is assumed to be at least twice differentiable and also ranges from 0 to 1. Within Eqn. (2.7), governed by the transition probability function $G_2$, the
expected duration process moves between two expected duration processes, $\psi_{(1)}^\delta$ and $\psi_{(2)}^\delta$, which are also governed by price changes. Accordingly, the specification of Eqn. (2.7) contains features that permits not only price changes but also past durations to influence the expected duration process. The above model setting adopts the framework of the multiple-regime smooth transition autoregressive model of van Dijk and Franses (1999), which is an extension of the basic smooth transition autoregressive model (STAR) popularized by Chan and Tong (1986), Granger and Teräsvirta (1993), and Teräsvirta (1994). Since the expected duration process described above considers price changes and past duration effects in a smooth transition approach, the model is labeled the smooth transition autoregressive conditional duration model of price and duration (STACD-PD).

The STACD-PD model presented here has similar features to the threshold autoregressive conditional duration (TACD) model of Zhang et al. (2001), the asymmetric ACD model of Bauwens and Giot (2003) and the ACM-ACD model of Russell and Engle (2005), but differs from these models in some respects. First, the past duration effect is considered in the model of Zhang et al. (2001) and the STACD-PD examined here. Zhang et al. (2001) proposed a threshold model for the duration process, termed the threshold ACD (TACD) model, by incorporating the duration of the previous transaction as a threshold variable. Consequently, the structural change pattern of the TACD model exhibits abrupt changes over distinct duration regimes. On the other hand, the past duration effect on the duration process considered in the STACD-PD model displays a smooth pattern. Unlike the threshold discontinuity in the TACD model, the structural change pattern of the duration in the STACD-PD model is determined by the duration data regarding whether the changes are instantaneous. Second, the effect of price change on the duration is considered in the asymmetric ACD model of Bauwens and Giot (2003), the ACM-ACD model of Russell and Engle (2005) and the STACD-PD model presented here. Bauwens and Giot (2003) considered the price trend effect on the duration process by employing the two-state transition probability matrix to model the price change directions, i.e., price increase and price decrease. Furthermore, Russell and Engle (2005) considered the multiple states of price changes and modeled the price change as a multinomial process termed the autoregressive
conditional multinomial model (ACM). 5 The models of both Bauwens and Giot (2003) and Russell and Engle (2005) use the joint distribution of the duration and the price change to estimate the impact of price change on the duration process. Although the joint distribution approach theoretically can be adapted to numerous states, determining the number of states in terms of empirical applications is highly subjective. Unlike the joint distribution approach to price change effect on the duration process in the models of Bauwens and Giot (2003) and Russell and Engle (2005), the STACD-PD model does not treat price change as a distribution process. Instead, the price change effect on the duration process adopts a smooth transition to affect the coefficients in the STACD-PD model similar to the past duration effect on the duration process in the STACD-PD model. Consequently, the STACD-PD model permits the price change data to determine the number of states in the smooth transition function and the adjustment speed of the structural change of the duration process in response to price changes.

For estimating the STACD-PD model, this study adopts the maximum likelihood estimation approach. The Weibull distribution is adopted as the distribution assumption. 6 Accordingly, the associated conditional intensity process, \( \lambda \), can be represented as follows (Engle and Russell, 1998):

\[
\lambda(t | x_1, x_2, ..., x_i) = \left( \Gamma\left(1 + \frac{1}{r}\right) \Theta_i \right) \left( t - t_i \right)^{-r} r,
\]

(2.9)

where \( \Gamma(\cdot) \) denotes the gamma function and \( r \) represents the Weibull distribution parameter that governs the shape of the conditional intensity process. Consequently, the density function under the Weibull distribution can be expressed as follows:

\[
f(x_i | \Omega_{i-1}, \Theta) = \frac{r}{x_i} \left( \frac{x_i \Gamma(1 + \frac{1}{r})}{\Theta_i} \right)^r \exp \left( - \left( \frac{x_i \Gamma(1 + \frac{1}{r})}{\Theta_i} \right) \right).
\]

(2.10)

5 Rather than focusing on the duration process given the price change in Bauwens and Giot (2003), the ACM-ACD model of Russell and Engle (2005) focuses on the price change process given the past duration.

6 The Weibull distribution assumption is used for the following reasons: first, the Weibull distribution has been widely used in the literature, such as Engle and Russell (1997, 1998), Hamelink (1998), and Bauwens and Giot (2000, 2003); second, it incorporates the feature of a non-constant conditional trading intensity function.
Furthermore, if the location parameter of the Weibull distribution is set to zero and the scale parameter is set to equal to \( \Theta_i \), the density function becomes (Hamelink, 1998, Lee, 1992):

\[
f(x_i | \Omega_{r-1}, \Theta) = \frac{r}{\Theta_i} \left( \frac{x_i}{\Theta_i} \right)^{r-1} \exp \left( - \left( \frac{x_i}{\Theta_i} \right)^r \right). \tag{2.11}
\]

Consequently, the conditional intensity process becomes:

\[
\lambda(t | x_1, x_2, ..., x_i) = \Theta_i^{-r}(t-t_i)^{-r}r. \tag{2.12}
\]

Following Eqn. (2.11), the log-likelihood function for the STACD-PD model can be written as follows:

\[
\ln L = \sum_{i=1}^{N} \left[ \log r - \log \Theta_i + (r-1)\log \left( \frac{x_i}{\Theta_i} \right) - \left( \frac{x_i}{\Theta_i} \right)^r \right]. \tag{2.13}
\]

### 3 Specification Test for the STACD-PD Model

To test the suitability of the STACD-PD compared to a regular Log-ACD, this study adopts the following approach, in which testing for the price change effect (or the past duration effect) is performed first\(^7\) after which the other effect is examined. Notably, the trade duration process \( \Psi_m \) for a given regime \( m \) can be expressed as follows:

\[
\Psi_m = w_m + \sum_{j=1}^{d} \alpha_m \epsilon_{i-j} + \sum_{k=1}^{d} \beta_m \psi_{i-k} = \Pi'_m Y_i, \quad m = 1, 2, 3, 4, \tag{3.1}
\]

\[
\Pi_m = [w_m, \alpha_m, \ldots, \alpha_m, \beta_m, \ldots, \beta_m]',
\]

\[
Y_i = [1, \epsilon_{i-1}, \ldots, \epsilon_{i-p}, \psi_{i-1}, \ldots, \psi_{i-q}]'.
\]

To test whether the price change influences the trade duration process, Eqn.(2.5) is rearranged as follows:

---

\(^7\) In practice, a first step can be conducted to test the existence of the price change and duration effects. Given these two effects, the model with the most significant effect can be estimated first.
where \( G'_i(\delta_{i-d_i}; \gamma_i, c_i) = G_i(\delta_{i-d_i}; \gamma_i, c_i) - 1/2 \). Replacing the transition function \( G'_i(\delta_{i-d_i}; \gamma_i, c_i) \) with a third-order Taylor expansion at \( \gamma_i = 0 \) for Eqn. (3.2), Eqn. (3.2) can be rearranged as follows:

\[
\psi_{(i)}^{\delta} = \lambda_0^\delta + \Phi'_\delta Y_{i1} + \varsigma_1 Y_{i2,\delta_{i-d_1}} + \varsigma_2 Y_{i2,\delta_{i-d_1}^2} + \varsigma_3 Y_{i2,\delta_{i-d_1}^3},
\]

where \( \lambda_0^\delta \) denotes the intercept. This transformation can avoid the problem of Davies (1987), namely the existence of unidentified nuisance parameters under the null hypothesis. This method is proposed by Luukkonen et al. (1988). Following the transformation, the null hypothesis \( H_0: \gamma_i = 0 \) can be restated as \( H_0: \varsigma_1 = \varsigma_2 = \varsigma_3 = 0 \). Consequently, using Eqn. (3.3) above and plugging it into the log-likelihood function of Eqn. (2.13), a test statistic of the Lagrange multiplier (LM) type, \( LM_{pc} \), is obtained for no price change effect, which can be stated via the following test procedure:

**Procedure 1:** The LM-type test for no price change effect on the STACD-PD model can be expressed as:

\[
LM_{pc} = S_\delta H_\delta^{-1} S_\delta \Rightarrow \chi^2(3(p + q + 1)),
\]

where \( \Rightarrow \) denotes the convergence in distribution, \( \Xi \) is the parameter set, and \( S_\delta \) and \( H_\delta \) are as follows:

\[
S_\delta = \left. \frac{\partial \ln L}{\partial \Xi} \right|_{\mu_0},
\]

\[
H_\delta = -E \left[ \frac{\partial^2 \ln L}{\partial \Xi \partial \Xi} \right]_{\mu_0}.
\]
The $LM_{pc}$ test above involves the first and second derivatives, which may not be applicable when data expose some irregularities. On the other hand, if the parameter $r$ of the Weibull distribution remains fixed, the first derivative of the log-likelihood function for each observation $i$ under the null hypothesis can be calculated as follows:

$$\frac{\partial l_i}{\partial \theta} \bigg|_{\mu_0} = \zeta_i^\delta Y_i,$$  

(3.4)

$$\frac{\partial l_i}{\partial \xi_j} \bigg|_{\mu_0} = \zeta_i^\delta Y_i \delta_{i-d1}^j, j = 1, 2, 3,$$  

(3.5)

$$\zeta_i^\delta = -r + r \left( \frac{x_i}{\Theta} \right)^\gamma.$$  

(3.6)

As suggested by Luukkonen et al. (1988) and van Dijk and Franses (1999), the $LM$-type test statistic for testing the null hypothesis of no price change effect can be conducted as follows.

**Procedure 2:** The following procedure can be adopted to establish the $\bar{LM}_{pc}$ to test for no price change effect in the STACD-PD model:

$$\bar{LM}_{pc} = \frac{(SSR_0^\delta - SSR_1^\delta)}{(p+q+1)} \Rightarrow F(3(p+q+1), (T-4(p+q+1))),$$

where $SSR_0$ and $SSR_1$ are calculated using the following steps.

1. Estimate the STACD-PD model using Eqn. (2.4) as the expected duration model, and the associated $\hat{\zeta}^\delta_i$ in Eqn. (3.6) are calculated. Then calculate the $SSR_0^\delta = \sum_{i=1}^{n} \hat{\zeta}^\delta_i$.
2. Regress $\hat{\zeta}^\delta_i$ on $Y_i$ and $\delta_{i-d1}^j, j = 1, 2, 3$ to obtain residuals $\tilde{\zeta}_i^\delta$, and calculate $SSR_1^\delta = \sum_{i=1}^{n} \tilde{\zeta}_i^\delta$.

**Remark 1:** To test for no past duration effect on the STACD-PD model, $\delta_{i-d1}$ is changed to $x_i-d_2$.

To test the existence of the subsequent duration effect, the same procedure is used as for testing the price change effect. First, Eqn. (2.7) is rewritten as follows:
Following replacing $G_s(x_{i-d}; \gamma_2, c_2)$ with a third-order Taylor expansion around $\gamma_2 = 0$, the model becomes

\[
\psi_i^* = \psi_1^* + \psi_2^* G_2(\delta_{i-d}; \gamma_1, c_1) + \psi_3^* G_2(x_{i-d}; \gamma_2, c_2)
+ \psi_4^* G_2(\delta_{i-d}; \gamma_1, c_1) G_2(x_{i-d}; \gamma_2, c_2) 
\]

(3.7)

\[
\psi_1^* = \psi_1, \\
\psi_2^* = \psi_2 - \psi_1, \\
\psi_3^* = \psi_3 - \psi_1, \\
\psi_4^* = \psi_4 - \psi_2 - \psi_3 + \psi_4. 
\]


Procedure 3: The LM-type test for no duration effects on the STACD-PD model can be expressed as:

\[
LM^{(pc)} = S_p H_D^{-1} S_D \Rightarrow \chi^2(6(p + q + 1)),
\]

where $\Xi$ is the parameter set and $S_D$ and $H_D$ are as follows:

\[
S_D = \frac{\partial \ln L}{\partial \Xi} \bigg|_{\mu_0},
\]
\[
H_D = -E \left[ \frac{\partial^2 \ln L}{\partial \Phi_1} \right]_{\mu_0}.
\]

Like the \(LM_{de}^{(pc)}\) test above, the first and second derivatives in \(LM_{de}^{(pc)}\) may not be applicable when the data displays irregularities. Accordingly, if parameter \(r\) of the Weibull distribution is fixed, the first derivative of the log-likelihood function for each observation \(i\) under the null hypothesis of no duration effect is as follows:

\[
\frac{\partial l_i}{\partial \Phi_1} \bigg|_{\mu_0} = \xi_i Y_i, \tag{3.9}
\]

\[
\frac{\partial l_i}{\partial \Phi_2} \bigg|_{\mu_0} = \xi_i D Y_i G_1(\delta_{x_{i-1}} \gamma_1, c_i), \tag{3.10}
\]

\[
\frac{\partial l_i}{\partial \gamma_j} \bigg|_{\mu_0} = \xi_i D Y_i x_{i-d2}, j = 1, 2, 3, \tag{3.11}
\]

\[
\frac{\partial l_i}{\partial \kappa_j} \bigg|_{\mu_0} = \xi_i D Y_i G_i(\delta_{x_{i-1}} \gamma_1, c_i)x_{i-d-2}, j = 4, 5, 6, \tag{3.12}
\]

\[
\frac{\partial l_i}{\partial \gamma_i} \bigg|_{\mu_0} = \xi_i D Y_i G_i(\delta_{x_{i-1}} \gamma_1, c_i) \frac{\partial G_i(\delta_{x_{i-1}} \gamma_1, c_i)}{\partial \gamma_i}, \tag{3.13}
\]

\[
\frac{\partial l_i}{\partial c_i} \bigg|_{\mu_0} = \xi_i D Y_i G_i(\delta_{x_{i-1}} \gamma_1, c_i) \frac{\partial G_i(\delta_{x_{i-1}} \gamma_1, c_i)}{\partial c_i}, \tag{3.14}
\]

\[
\xi_i^D = -r + r \left( \frac{x_i}{\Theta} \right), \tag{3.15}
\]

\[
\frac{\partial G_{\hat{h}}}{\partial \gamma_i} = G_{\hat{h} \gamma_i} = G_i[1 - G_i](\delta_{x_{i-1}} - \hat{c}_i), \tag{3.16}
\]

\[
\frac{\partial G_{\hat{h}}}{\partial c_i} = G_{\hat{h} c_i} = \gamma_i G_i[1 - G_i]. \tag{3.17}
\]

Consequently, the \(LM\)-type test statistic for testing the null hypothesis of no subsequent duration effect can be conducted as follows:
Procedure 4: The following procedure can be taken to form the $\bar{LM}_{d(c)(1)}^{(pc)}$:

$$\bar{LM}_{d(c)(1)}^{(pc)} = \frac{(SSR_0^D - SSR_1^D)}{SSR_1^D} \cdot \frac{6(p + q + 1)}{(T - 8(p + q + 1))} \Rightarrow F(6(p + q + 1), (T - 8(p + q + 1))) ,$$

where $SSR_0^D$ and $SSR_1^D$ are calculated as follows.

1. Estimate the STACD-PD model using Eqn. (2.5) as the expected duration model, and calculate the associated $\hat{\delta}_i$ in Eqn. (3.15). Then calculate the $SSR_0^D = \sum_{i=1}^{N} \hat{\delta}_i^D$.

2. Regress $\hat{\delta}_i$ on $Y_i$, $Y_{\gamma_i}$, $\Phi_i Y_{\gamma_i}$, $\Phi_i' Y_{\gamma_i}$, $Y_{\gamma_i}(j=1,2,3)$, and $Y_{\gamma_i} x_{i-\tau}$ ($j = 4,5,6$) to obtain residuals $\hat{\zeta}_i^D$, and calculate the $SSR_1^D = \sum_{i=1}^{N} \hat{\zeta}_i^D$.

Remark 2: If the $\gamma_1$ is excessively large, the partial derivatives of $G_{i1}$ with respect to $\gamma_1$ and $c_1$ are close to zero. Thus, regressors containing the terms $G_{i1}$ and $G_{i1}\gamma_1$ cannot provide any useful information contributing to the test statistics. In this situation, the $SSR_0^D = \sum_{i=1}^{N} \hat{\zeta}_i^D$ can be calculated, where $\hat{\zeta}_i^D$ are residuals obtained by regressing $\hat{\delta}_i$ on $Y_i$ and $Y_{\gamma_i}$ (or $\Phi_i' Y_{\gamma_i}$ and $\Phi_i' Y_{\gamma_i}$ may be included). The $SSR_1^D = \sum_{i=1}^{N} \hat{\zeta}_i^D$ is then calculated, where $\hat{\zeta}_i^D$ are residuals obtained by regressing $\hat{\delta}_i$ on $Y_{\gamma_i}(j=1,2,3)$ and $Y_{\gamma_i} x_{i-\tau}(j = 4,5,6)$. This statistic is denoted as $\bar{LM}_{d(c)(2)}$.

Remark 3: If the model is first estimated using the past duration effect, the existence of the subsequent price change effect can be tested by forming $\bar{LM}_{d(c)(1)}^{(dc)}$ and $\bar{LM}_{d(c)(2)}^{(dc)}$ which resemble $\bar{LM}_{d(c)(1)}^{(pc)}$ and $\bar{LM}_{d(c)(2)}^{(pc)}$ but with $x_{i-d}$ substituted for $x_{i-d1}$.

4 Empirical Application to IBM Stock

This section applies the STACD-PD model to trade durations of IBM stock to illustrate the empirical implementation of the STACD-PD model. First, this section presents the data description, and then it presents the results of the estimation and specification tests. Next, the data is classified into different regimes according to price changes and past trade durations. Finally, the moments, generalized impulse responses, and conditional trading intensity of regimes are discussed.
4.1 Data Description

The empirical data comprise intraday trades and quotes of IBM stock traded on the New York Stock Exchange (NYSE). The data were obtained from the Trades and Quotes (TAQ) database, compiled and made available by the NYSE. The transaction data were retrieved from the consolidated trade file, while the quote data were retrieved from the consolidated quote file. The sample period covers the entire month of May 2001. May was selected because firms’ financial quarterly reports are primarily announced and made public in April, a phenomenon that may influence regular investor trading behavior during this period. The data are divided into two distinct data sets. The first data set is designed to examine duration behavior during the opening hours of the stock market. Therefore, the first data set includes observations from 9:30 to 10:00 A.M.8 Meanwhile, the second data set is designed to compare the empirical results obtained in this study with those in the literature, and includes observations from 10:00 A.M. to 4:00 P.M.9

Table 1 summarizes the statistics of the trade durations in the sample. The sample includes 6,276 transactions in the opening hours from 9:30 to 10:00 A.M. and 68,801 transactions in the trading hours from 10:00 A.M. to 4:00 P.M., each with a distinct trading time. As in Engle and Russell (1998), Bauwens and Giot (2003), and Zhang et al. (2001), it is important to de-seasonalize raw trade durations before estimating the model due to the trading structure of the exchange and the trading behavior of traders and market makers. The seasonally adjusted trade durations assume a multiplicative form: \[ x_j^* = x_j / \phi(t_j) \], where \( x_j \) denotes the raw trade duration, \( \phi(t_j) \) represents the seasonal component, and \( x_j^* \) is the seasonally adjusted trade duration. As in Zhang et al. (2001), the seasonal component data, \( \phi(t_j) \), are obtained using the super smoother method proposed by Friedman (1984), where the local cross-validation is used to identify the span of data. For both data sets, the mean and standard deviation of the seasonally adjusted trade durations are smaller than those of raw trade durations, while the skewness and kurtosis of seasonally adjusted trade durations are close to those of raw trade durations. Furthermore, the autocorrelation for seasonally adjusted trade durations is still present as in the raw trade durations.

---

8 We appreciate the editor’s comment regarding this point.
9 A five-minute reporting delay is allowed.
Table 1. Descriptive Statistics of IBM Trade Durations

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Skewness</th>
<th>Kurtosis</th>
<th>LB(15)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A. Opening Hours:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Original Duration</td>
<td>6.2057</td>
<td>5.4354</td>
<td>3.1760</td>
<td>28.2724</td>
<td>360.08***</td>
</tr>
<tr>
<td>De-seasonalized Duration</td>
<td>1.0869</td>
<td>0.9581</td>
<td>3.2920</td>
<td>30.9888</td>
<td>398.61***</td>
</tr>
<tr>
<td><strong>B. Trading Hours:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Original duration</td>
<td>6.9276</td>
<td>6.6041</td>
<td>2.8397</td>
<td>16.5728</td>
<td>6,142.00***</td>
</tr>
<tr>
<td>De-seasonalized Duration</td>
<td>1.0426</td>
<td>0.9627</td>
<td>2.7030</td>
<td>15.5992</td>
<td>3,343.30***</td>
</tr>
</tbody>
</table>

a. The opening hours are from 9:30 to 10:00 A.M. and the trading hours are from 10:00 A.M. to 4:00 P.M.
b. The numbers of observations are 6,276 for the opening hours and 68,801 for the trading hours. The unit is seconds.
c. The Ljung-Box statistics of 15 lags are reported in LB(15).
d. *, **, and *** represent significance levels at 10%, 5%, and 1%, respectively.

4.2 Estimation Results of the STACD-PD Model

Table 2 lists the summary statistics for various specifications without price change and past duration during both the opening and trading hours. Clearly, the specification with three lags of both unexpected and expected trade durations has significantly higher log-likelihood values and lower AIC and Schwarz values than other specifications. As a result, the model with three lags of both unexpected and expected trade durations is adopted below.

Table 3 lists the estimation results obtained using the STACD-PD model for the opening hours. Panel A of Table 3 describes the estimation results without price change and past duration. The coefficients are significant with the exception of $\beta_{12}$. Moreover, the residuals do not exhibit any serious autocorrelation and are not overdispersed. However, the specification tests indicate the existence of the past duration effect but no price change effect. Meanwhile, the $p$-values in the specification tests suggest that the lag one trade duration, $x_{t-1}$, has the lowest $p$-value. Therefore, the lag one trade duration, $x_{t-1}$, is included as the transition variable and the model is estimated with the past duration effect for the opening hours. Panel B of Table 3 lists the estimation results of the model with the past duration effect.
Table 2. Summary Statistics for Various Specifications without Price Change and Duration Effects

<table>
<thead>
<tr>
<th>Model</th>
<th>Log-Likelihood</th>
<th>AIC</th>
<th>Schwarz</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. Opening Hours:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1,1)</td>
<td>-6,271.73</td>
<td>2.000232</td>
<td>2.004531</td>
</tr>
<tr>
<td>(2,2)</td>
<td>-6,263.03***</td>
<td>1.998415</td>
<td>2.004864</td>
</tr>
<tr>
<td>(3,3)</td>
<td>-6,253.66***</td>
<td>1.996383</td>
<td>2.004984</td>
</tr>
<tr>
<td>(4,4)</td>
<td>-6,255.42</td>
<td>1.997900</td>
<td>2.008653</td>
</tr>
<tr>
<td>B. Trading Hours:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1,1)</td>
<td>-67,325.76</td>
<td>1.957260</td>
<td>1.957792</td>
</tr>
<tr>
<td>(2,2)</td>
<td>-67,200.25***</td>
<td>1.953698</td>
<td>1.954495</td>
</tr>
<tr>
<td>(3,3)</td>
<td>-67,193.01***</td>
<td>1.953574</td>
<td>1.954637</td>
</tr>
<tr>
<td>(4,4)</td>
<td>-67,192.37</td>
<td>1.953642</td>
<td>1.954971</td>
</tr>
</tbody>
</table>

a. The opening hours are from 9:30 to 10:00 A.M. and the trading hours are from 10:00 A.M. to 4:00 P.M.
b. The numbers of observations are 6,276 for the opening hours and 68,801 for the trading hours.
c. The (p,q) in the model column stands for p lags of $\epsilon_t$ and q lags of $\psi_t$.
d. *, **, and *** represent significance levels at 10%, 5%, and 1%, respectively, of the likelihood ratio test of that model against the preceding specification. The degrees of freedom are 2.

duration effect for the opening hours. The estimated model is found to have good fitness since the residuals are free of autocorrelation and overdispersion. The specification tests indicate that there is no subsequent price change effect. Notably, the coefficients of $\Psi_2$ become more influential with longer lagged trade duration. The significantly positive $\alpha_{21}$ implies that the current expected trade duration becomes elongated given the existence of an unexpectedly long lagged trade duration in the regime of the longer past trade duration, $x_{t-1}$. Additionally, the significant $\beta_{23}$ in $\Psi_2$ and insignificant $\beta_{13}$ in $\Psi_1$ may imply that the expected trade duration appears considerably more persistent in the regime with the longer past trade duration. Consequently, during the opening hours, the past duration effect influences the structural change behavior of the expected trade durations while the price change effect exerts no such influence. Moreover, the expected trade durations become more persistent given longer past trade duration. These findings may highlight the phenomenon of numerous liquidity traders.
Table 3. Estimation Results of the STACD-PD Model for the Opening Hours of the IBM Stock

Panel A: without price change and duration effects

<table>
<thead>
<tr>
<th>Model</th>
<th>$w_1$</th>
<th>$\alpha_{11}$</th>
<th>$\alpha_{12}$</th>
<th>$\alpha_{13}$</th>
<th>$\beta_{11}$</th>
<th>$\beta_{12}$</th>
<th>$\beta_{13}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Psi_1$</td>
<td>-0.0098</td>
<td>0.0384</td>
<td>0.0539</td>
<td>-0.0809</td>
<td>1.2106</td>
<td>0.2117</td>
<td>-0.4260</td>
</tr>
<tr>
<td></td>
<td>(0.0037)**</td>
<td>(0.0065)**</td>
<td>(0.0133)**</td>
<td>(0.0102)**</td>
<td>(0.1429)**</td>
<td>(0.2605)</td>
<td>(0.1350)**</td>
</tr>
</tbody>
</table>

Weibull $r$

1.3429

(0.0124)***

<table>
<thead>
<tr>
<th>Specification Tests</th>
<th>Tests of Residuals</th>
<th>Dispersion</th>
<th>$LB(15)$</th>
<th>$LB^2(15)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test</td>
<td>$\hat{d}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1.0296</td>
<td>1.2337</td>
<td>1.3261</td>
<td>0.8471</td>
</tr>
<tr>
<td>2</td>
<td>[0.4215]</td>
<td>[0.2106]</td>
<td>[0.1451]</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Panel B: with the duration effect

<table>
<thead>
<tr>
<th>Model</th>
<th>$w_1$</th>
<th>$\alpha_{11}$</th>
<th>$\alpha_{12}$</th>
<th>$\alpha_{13}$</th>
<th>$\beta_{11}$</th>
<th>$\beta_{12}$</th>
<th>$\beta_{13}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Psi_1$</td>
<td>-0.0046</td>
<td>0.0160</td>
<td>0.0604</td>
<td>-0.0709</td>
<td>1.9194</td>
<td>-1.0243</td>
<td>0.1032</td>
</tr>
<tr>
<td></td>
<td>(0.0017)**</td>
<td>(0.0111)</td>
<td>(0.0227)**</td>
<td>(0.0139)**</td>
<td>(0.1379)**</td>
<td>(0.2708)**</td>
<td>(0.156)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\Psi_2$</th>
<th>$w_1$</th>
<th>$\alpha_{21}$</th>
<th>$\alpha_{22}$</th>
<th>$\alpha_{23}$</th>
<th>$\beta_{21}$</th>
<th>$\beta_{22}$</th>
<th>$\beta_{23}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-0.0158</td>
<td>0.0938</td>
<td>-0.0142</td>
<td>-0.0076</td>
<td>2.0641</td>
<td>-1.8936</td>
<td>0.8252</td>
</tr>
<tr>
<td></td>
<td>(0.0032)**</td>
<td>(0.0106)**</td>
<td>(0.0271)</td>
<td>(0.0199)</td>
<td>(0.0900)**</td>
<td>(0.1620)**</td>
<td>(0.0830)**</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\gamma_2$</th>
<th>$c_2$</th>
<th>Weibull $r$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>49.0384</td>
<td>1.4652</td>
</tr>
<tr>
<td></td>
<td>[106.8585]</td>
<td>(0.0603)**</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Specification Tests</th>
<th>Tests of Residuals</th>
<th>Dispersion</th>
<th>$LB(15)$</th>
<th>$LB^2(15)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test</td>
<td>$\hat{d}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1.1938</td>
<td>1.2069</td>
<td>1.3990</td>
<td>0.8435</td>
</tr>
<tr>
<td>2</td>
<td>[0.1829]</td>
<td>[0.1692]</td>
<td>[0.0707]*</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$\tilde{LM}_{pc}^{(a)}$

| | | |
| | | |

$\tilde{LM}_{rk}^{(a)}$

| | | |
| | | |

Note:

a. The opening hours are from 9:30 to 10:00 A.M. and the number of observations is 6,276.

b. The estimated model is given with $p = q = 3$.
Table 3. (continued)

\[ \Psi_j = \Psi_{ij} (1 - G_j^i) + \Psi_{ij} G_j^i \]

\[ \Psi_{ij} = \omega_m + \sum_{j=1}^{5} a_{ij} \psi_{i}^{j-1} + \sum_{k=1}^{5} \beta_{ij} \psi_{ij+k}^j \cdot m=1,2 \cdot G_j^{\text{exp}(\psi_{i,j-1}, \psi_{i,j+2})} \cdot \frac{1}{1 + \exp(\psi_{i,j-1} - \psi_{i,j+2})} \]

c. \( \hat{LM}_p \) and \( \hat{LM}_p \) are test statistics for the price change and duration effects, respectively. \( \hat{LM}_{p,1}^{\text{ad}} \) and \( \hat{LM}_{p,1}^{\text{ad}} \) are test statistics for the subsequent price change effect given the duration effect.

d. The numbers in the parentheses are the standard errors and the numbers in the brackets are the p-values.

e. Dispersion is the variation coefficient of residuals.

f. The Ljung-Box Statistics of 15 lags for residuals and squared residuals are reported in \( LB(15) \) and \( LB'(15) \), respectively.

g. *, **, and *** represent significance levels at 10%, 5%, and 1%, respectively.

Russell and Engle (2005) found that declining past prices are predictive of longer expected trade durations. The findings here demonstrate that the existence of clustering and trading with one another during the opening hours, reducing the correlation of trade durations with price variation.\(^{11}\)

Table 4 lists estimation results of the STACD-PD model for the trading hours. The specification tests in Panel A of Table 4 reveal the existence of price change and past duration effects, although residuals do not display any serious autocorrelation and overdispersion. It is found that lag one price change has the smallest p-value in the specification tests, and is considered as the transition variable. Consequently, the model with the price change effect is subsequently estimated and listed in Panel B of Table 4. The tests of subsequent past duration effect recommend the existence of the subsequent past duration effect, and the lag two duration has the smallest p-value. Incorporating the lag two duration as the additional transition variable, the model with price change and past duration effects is estimated and shown in Panel C of Table 4. The insignificant coefficient estimates in \( \Psi_3 \) indicate that expected trade durations are not dependent on past expected and unexpected trade durations given the declining past price and the longer past duration.

\(^{11}\) Admati and Pfleiderer (1988) demonstrated that liquidity traders tend to be concentrated in trades on the opening, which involve larger trading volumes. Chiang et al. (2006) also found that the correlation between trade durations and price volatility becomes insignificant given large numbers of liquidity trades.
Table 4. Estimation Results of the STACD-PD Model for the Trading Hours of the IBM Stock

Panel A: without price change and duration effects

<table>
<thead>
<tr>
<th>Model</th>
<th>( w_1 )</th>
<th>( \alpha_{11} )</th>
<th>( \alpha_{12} )</th>
<th>( \alpha_{13} )</th>
<th>( \beta_{11} )</th>
<th>( \beta_{12} )</th>
<th>( \beta_{13} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Psi_1 )</td>
<td>-0.0003</td>
<td>0.0503</td>
<td>-0.0849</td>
<td>0.0350</td>
<td>2.5718</td>
<td>-2.1737</td>
<td>0.6019</td>
</tr>
<tr>
<td></td>
<td>(0.0001)***</td>
<td>(0.0025)***</td>
<td>(0.0054)***</td>
<td>(0.0039)***</td>
<td>(0.0895)***</td>
<td>(0.1710)***</td>
<td>(0.0816)***</td>
</tr>
</tbody>
</table>

Weibull \( r \)

1.2767

(0.0035)***

<table>
<thead>
<tr>
<th>Specification Tests</th>
<th>Tests of Residuals</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test</td>
<td>( d )</td>
</tr>
<tr>
<td>1</td>
<td>2.9814</td>
</tr>
<tr>
<td>2</td>
<td>1.4969</td>
</tr>
</tbody>
</table>

Panel B: with the price change effect

<table>
<thead>
<tr>
<th>Model</th>
<th>( w_1 )</th>
<th>( \alpha_{11} )</th>
<th>( \alpha_{12} )</th>
<th>( \alpha_{13} )</th>
<th>( \beta_{11} )</th>
<th>( \beta_{12} )</th>
<th>( \beta_{13} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Psi_1 )</td>
<td>0.0902</td>
<td>-0.1603</td>
<td>0.2719</td>
<td>-0.1838</td>
<td>1.8452</td>
<td>-1.4903</td>
<td>0.3442</td>
</tr>
<tr>
<td></td>
<td>(0.0304)***</td>
<td>(0.0612)***</td>
<td>(0.0972)***</td>
<td>(0.0650)***</td>
<td>(0.1690)***</td>
<td>(0.2517)***</td>
<td>(0.1642)***</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( \Psi_2 )</th>
<th>( w_2 )</th>
<th>( \alpha_{21} )</th>
<th>( \alpha_{22} )</th>
<th>( \alpha_{23} )</th>
<th>( \beta_{21} )</th>
<th>( \beta_{22} )</th>
<th>( \beta_{23} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-0.0004</td>
<td>0.0631</td>
<td>-0.1060</td>
<td>0.0434</td>
<td>2.5145</td>
<td>-2.0644</td>
<td>0.5498</td>
</tr>
<tr>
<td></td>
<td>(0.0001)***</td>
<td>(0.0054)***</td>
<td>(0.0094)***</td>
<td>(0.0053)***</td>
<td>(0.0985)***</td>
<td>(0.1877)***</td>
<td>(0.0892)***</td>
</tr>
</tbody>
</table>

Weibull \( r \)

20.0564

(3.9068)***

(0.0206)***

(0.0035)***

<table>
<thead>
<tr>
<th>Specification Tests</th>
<th>Tests of Residuals</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test</td>
<td>( d )</td>
</tr>
<tr>
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<td>2.3562</td>
</tr>
<tr>
<td>2</td>
<td>0.9496</td>
</tr>
</tbody>
</table>
Table 4. (continued)
Panel C: with the price change effect and the subsequent duration effect

<table>
<thead>
<tr>
<th>Model</th>
<th>$w_1$</th>
<th>$\alpha_{11}$</th>
<th>$\alpha_{12}$</th>
<th>$\alpha_{13}$</th>
<th>$\beta_{11}$</th>
<th>$\beta_{12}$</th>
<th>$\beta_{13}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Psi_1$</td>
<td>0.0725</td>
<td>-0.1281</td>
<td>-0.0842</td>
<td>0.1356</td>
<td>0.9726</td>
<td>-0.5676</td>
<td>0.5904</td>
</tr>
<tr>
<td></td>
<td>(0.0504)</td>
<td>(0.0476)**</td>
<td>(0.0969)</td>
<td>(0.0856)</td>
<td>(0.1843)**</td>
<td>(0.2622)**</td>
<td>(0.1557)**</td>
</tr>
<tr>
<td>$w_2$</td>
<td>-0.0004</td>
<td>0.0567</td>
<td>-0.0940</td>
<td>0.0377</td>
<td>2.5518</td>
<td>-2.1345</td>
<td>0.5826</td>
</tr>
<tr>
<td></td>
<td>(0.0001)**</td>
<td>(0.0043)**</td>
<td>(0.0082)**</td>
<td>(0.0053)**</td>
<td>(0.0093)**</td>
<td>(0.1906)**</td>
<td>(0.0914)**</td>
</tr>
<tr>
<td>$w_3$</td>
<td>0.1396</td>
<td>-0.0550</td>
<td>0.0394</td>
<td>-0.0486</td>
<td>1.2170</td>
<td>-1.1639</td>
<td>0.1562</td>
</tr>
<tr>
<td></td>
<td>(0.1101)</td>
<td>(0.0555)</td>
<td>(0.0579)</td>
<td>(0.0601)</td>
<td>(0.8377)</td>
<td>(0.8883)</td>
<td>(0.8359)</td>
</tr>
<tr>
<td>$w_4$</td>
<td>-0.0003</td>
<td>0.0514</td>
<td>-0.0890</td>
<td>0.0379</td>
<td>2.5330</td>
<td>-2.0979</td>
<td>0.5649</td>
</tr>
<tr>
<td></td>
<td>(0.0001)**</td>
<td>(0.0050)**</td>
<td>(0.0087)**</td>
<td>(0.0045)**</td>
<td>(0.1029)**</td>
<td>(0.1981)**</td>
<td>(0.0954)**</td>
</tr>
</tbody>
</table>

$\gamma_1$ | $c_1$ | $\gamma_2$ | $c_2$ | Weibull $r$

|        | 33.2300 | -0.1120 | 618.4198 | 1.0339 | 1.2778 |
|        | (5.4852)** | (0.1040)** | (4072.1290) | (0.0165)** | (0.0036)** |

Tests of Residuals

<table>
<thead>
<tr>
<th>Dispersion</th>
<th>$LB(15)$</th>
<th>$LB^*(15)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>17.6494</td>
<td>12.0293</td>
</tr>
<tr>
<td>(0.2815)</td>
<td>(0.6768)</td>
<td></td>
</tr>
</tbody>
</table>

a. The opening hours are from 10:00 A.M. to 4:00 P.M. and the number of observations is 68,801.
b. The estimated model is given with $p=q=3$:

$$
\gamma_i = (\Psi_i (1 - G) + \Psi_3 G) (1 - G) + (\Psi_4 (1 - G) + \Psi_3 G) G_i
$$

$$
\gamma_m = \gamma_m + \sum_{j=1}^{q} a_{mj} \psi_{i-j} + \sum_{k=1}^{r} b_{mk} \psi_{i-k}, \quad m=1, 2, 3, 4
$$

$$
G_i (\delta_{ij}; \gamma_i, c_i) = \frac{1}{1 + \exp(-\gamma_i (\delta_{ij} - c_i))}, \quad G_2 (\delta_{ij}; \gamma_2, c_2) = \frac{1}{1 + \exp(-\gamma_2 (\delta_{ij} - c_2))}
$$

c. $LM_p$ and $LM_d$ are test statistics for the price change and duration effects, respectively. $LM_{p(1)}$ and $LM_{d(1)}$ are test statistics for the subsequent duration effect given the price change effect.
d. The numbers in the parentheses are the standard errors and the numbers in the brackets are the $p$-values.
e. Dispersion is the variation coefficient of residuals.
f. The Ljung-Box Statistics of 15 lags for residuals and squared residuals are reported in $LB(15)$ and $LB^*(15)$, respectively.
g. *, **, and *** represent significance levels at 10%, 5%, and 1%, respectively.
of negative relationship between the expected trade duration and the past price change only given shorter past durations. Consequently, trading becomes infrequent once past trading is frequent due to shorter past trade durations and the price tends to decline. This phenomenon may result from the constraints on short selling \(^{12}\) following periods of reducing prices and the higher probability of trading against informed traders when past trading is frequent. \(^{13}\) However, we find that the magnitude of coefficient estimates in \(\Psi_2\) and \(\Psi_4\) associated with the price increase regime are larger than those in \(\Psi_1\) and \(\Psi_3\) associated with the price decrease regime. Combined with insignificant coefficient estimates in \(\Psi_3\), this demonstrates that expected trade durations are more persistent in the upward price regime, in contrast to the findings of Bauwens and Giot (2003). Bauwens and Giot (2003) found that expected duration is equally persistent for both price increase and price decrease regimes. The difference between the findings of this study and those of Bauwens and Giot (2003) may result from the fact that the model estimated here incorporates the past duration effect as the transition variable while Bauwens and Giot (2003) only consider the price change effect. As a result, the price change effect accompanied by the duration effect is able to identify the expected duration behavior in more detail.

### 4.3 Characteristics and Moments of Distinct Regimes

To further explore the implications of the STACD-PD model, the opening hours data are grouped into two distinct regimes according to lagged trade duration: (1) in the DG regime the past trade duration equals or exceeds the mean trade duration, and (2) in the DL regime the past trade duration is less than the mean trade duration. Consequently, the DG regime represents the longer trade duration regime, while the DL represents the shorter trade duration regime. Panel A of Table 5 illustrates the market microstructure characteristics of the four regimes. Spreads and trading volume per second in the DG regime significantly exceed those in the whole opening hours while the volatility per second in the DG regime

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12 Diamond and Verrecchia (1987) found that short sale constraints reduce the information adjustment speed during the downward market and increase the trade duration.

13 Easley and O’Hara (1992) posited that more frequent trading is related to the existence of more informed traders. Accordingly, they proposed that longer trade durations are associated with smaller price variation owing to infrequent trading.
Table 5. The Characteristics of the Opening Hours and Associated Two Regimes, and Estimation Results of the Logistic Regression

Panel A: The Characteristics of the Opening Hours and Associated Two Regimes

<table>
<thead>
<tr>
<th>Spread</th>
<th>Volume/s</th>
<th>Volatility/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>SE</td>
<td>Mean</td>
</tr>
<tr>
<td>Whole Opening Hours</td>
<td>0.0749</td>
<td>0.0490</td>
</tr>
<tr>
<td>DG</td>
<td>0.0800‡</td>
<td>0.0502</td>
</tr>
<tr>
<td>DL</td>
<td>0.0722</td>
<td>0.0481</td>
</tr>
</tbody>
</table>

Panel B: Estimation Results of the Logistic Regression with the DG as the Choice Regime

<table>
<thead>
<tr>
<th>Intercept</th>
<th>Spread</th>
<th>Volume/s</th>
<th>Volatility/s</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3.0487</td>
<td>0.1253</td>
<td>0.0065</td>
</tr>
</tbody>
</table>

a. Spread represents the difference between prevailing bid and ask prices. Volume/s represents the ratio of the de-seasonalized transaction volume divided by the de-seasonalized trade duration. Volatility/s represents the ratio of the absolute change in the midpoint of prevailing quote divided by the de-seasonalized trade duration.

b. Mean represents the average. SE represents the standard error. The standard errors of parameter estimates are in parentheses.

c. † and ‡ indicate that the number in the cell is significantly larger than of the whole data during opening hours at significance levels of 10% and 5%, respectively, under the Mann-Whitney-Wilcoxon mean test.

d. *, **, and *** represent significance levels at 10%, 5%, and 1%, respectively, when the corresponding mean is larger than of the whole data during opening hours.

e. (1) DG regime in which the lagged trade duration change equals or exceeds the mean of trade durations, and (2) the DL regime in which the lagged trade duration change is less than the mean of trade durations.

f. The numbers of observations are 6,276, 2,190, and 4,086 for the whole opening hours, DG, and DL, respectively.

g. The logistic regression is specified as follows:

\[
\log \left( \frac{u}{1-u} \right) = a + b \cdot \text{Spread}_i + b \cdot \text{Volume/s}_i + b \cdot \text{Volatility/s}_i
\]

where \( u \) = \( E(z_i) \) and \( z_i = 1 \) if the observation \( i \) belongs to the DG regime.

is not significantly larger than in the whole opening hours. Fundamentally, spreads comprise order processing costs, inventory costs, and information costs (for example, Stoll, 1989 and van Ness et al., 2001). The DG regime does not display serious information asymmetry because of the insignificant volatility per second, while inventory and order processing costs are relatively higher owing to the significant trading volume per second. The higher inventory and order processing costs create higher spreads in the DG regime. Panel B of Table 5 shows the estimation results of a Logit Model with the DG regime as the choice regime, and
also confirms that wider spreads and larger trading volume per second significantly increase the probability of occurrence of the DG regime. The trading hour data are also grouped into four different regimes based on past price changes and lag two trade durations: (1) in the PGDG regime the past price change equals or exceeds zero and the lag two trade duration equals or exceeds the mean of trade durations, (2) in the PLDG regime the past price change is less than zero and the lag two trade duration equals or exceeds the mean of trade durations, (3) in the PGDL regime the past price change equals or exceeds zero and the lag two trade duration is less than the mean trade duration, and (4) in the PLDL regime the past price change is less than zero and the lag two trade duration is less than the mean trade duration. The PGDG and PGDL regimes are upward price change regimes, and the PLDG and PLDL regimes are downward price change regimes. Meanwhile, the PGDG and PLDG regimes are longer trade duration regimes while the PGDL and PLDL are shorter trade duration regimes.

Panel A of Table 6 summarizes the market microstructure characteristics of the four regimes. Notably, compared to the whole trading hours data, the PLDG and PLDL regimes have significantly higher volatility per second. Consequently, the information asymmetry is more serious for the downward price regimes due to the higher volatility per second. Restated, when the price is trending downwards, the probability of trading against informed traders increases. The asymmetric findings for the upward and downward markets are consistent with a stylized fact in the finance literature. Generally, negative returns generate higher unexpected volatility than positive returns (for example, French et al., 1987, Schwert, 1989). Engle and Ng (1993) demonstrated that a news impact curve has an asymmetric pattern in which negative return shocks increase predictable volatility more than positive return shocks. The reason for this asymmetric pattern during the upward and downward trends could be due to the leverage effect\(^\text{14}\) from stock price declines increasing financial leverage, and thus increasing the risk of a company going bankrupt. Zhang et al. (2001) found that the fast transaction regime with shorter past trade durations can be characterized as an informed trading regime involving a high likelihood of trading against informed traders. This study

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\(^{14}\) Figlewski and Wang (2000) proposed that the leverage effect could be better labeled the downward market effect since their empirical evidence reveals only a weak direct relationship between the leverage effect and firm leverage.
### Table 6. The Characteristics of the Trading Hours and Associated Four Regimes, and Estimation Results of the Logistic Regression

#### Panel A: The Characteristics of the Trading Hours and Four Regimes

<table>
<thead>
<tr>
<th></th>
<th>Spread Mean</th>
<th>Spread SE</th>
<th>Volume/s Mean</th>
<th>Volume/s SE</th>
<th>Volatility/s Mean</th>
<th>Volatility/s SE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Whole Trading Hours</td>
<td>0.0603</td>
<td>0.0383</td>
<td>1.4401</td>
<td>3.0634</td>
<td>0.0153</td>
<td>0.0380</td>
</tr>
<tr>
<td>PGDG</td>
<td>0.0616†</td>
<td>0.0382</td>
<td>1.5246</td>
<td>3.0686</td>
<td>0.0139</td>
<td>0.0357</td>
</tr>
<tr>
<td>PLDG</td>
<td>0.0601</td>
<td>0.0380</td>
<td>1.4567</td>
<td>3.0243</td>
<td>0.0168‡</td>
<td>0.0421</td>
</tr>
<tr>
<td>PGDL</td>
<td>0.0607</td>
<td>0.0385</td>
<td>1.4434</td>
<td>3.1675</td>
<td>0.0150</td>
<td>0.0377</td>
</tr>
<tr>
<td>PLDL</td>
<td>0.0582</td>
<td>0.0381</td>
<td>1.3259</td>
<td>2.8304</td>
<td>0.0169‡</td>
<td>0.0390</td>
</tr>
</tbody>
</table>

#### Panel B: Estimation Results of the Logistic Regression with the PLDL as the Choice Regime

<table>
<thead>
<tr>
<th>Regime</th>
<th>Intercept</th>
<th>Spread</th>
<th>Volume/s</th>
<th>Volatility/s</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-1.2499</td>
<td>-1.9504</td>
<td>-0.0187</td>
<td>1.4814</td>
</tr>
<tr>
<td></td>
<td>(0.0182)***</td>
<td>(0.2521)***</td>
<td>(0.0036)***</td>
<td>(0.2313)***</td>
</tr>
</tbody>
</table>

a. Spread represents the difference between prevailing bid and ask prices. Volume/s represents the ratio of the de-seasonalized transaction volume divided by the de-seasonalized trade duration. Volatility/s represents the ratio of the absolute change in the midpoint of prevailing quote divided by the de-seasonalized trade duration.
b. Mean represents the average. SE represents the standard error. The standard errors of parameter estimates are in parentheses.
c. † and ‡ indicate that the number in the cell is significantly larger than of the whole data during trading hours at significance levels of 10% and 5%, respectively, under the Mann-Whitney-Wilcoxon mean test.
d. *, **, and *** represent significance levels at 10%, 5%, and 1%, respectively, when the corresponding mean is larger than of the whole data during trading hours.
e. (1) PGDG regime in which the lagged price change equals or exceeds zero and the lagged trade duration is equals or exceeds to the mean of trade durations, (2) the PLDG regime in which the lagged price change is less than zero and the lagged trade duration equals or exceeds the mean of trade durations, (3) the PGDL regime in which the lagged price change equals or exceeds zero and the lagged trade duration is less than the mean of trade durations, and (4) the PLDL regime in which the lagged price change is less than zero and the lagged trade duration is less than the mean of trade durations.
f. The numbers of observations are 68,801, 16,207, 7,508, 31,129, and 13,957 for the whole trading hours, PGDG, PLDG, PGDL, and PLDL, respectively.
g. The logistic regression is specified as follows:

\[
\log \left( \frac{u_i}{1 - u_i} \right) = a + b_1 \text{Spread}_i + b_2 \text{Volume/s}_i + b_3 \text{Volatility/s}_i
\]

where \( u_i = E(z_i) \) and \( z_i = 1 \) if the observation belongs to the PLDL regime.
complements the findings of Zhang et al. (2001) and indicates that informed trading regimes are associated with price declines and shorter past trade durations. The estimation results obtained via a Logit Model with the PLDL regime as the choice regime in Panel B of Table 6 also confirm that higher volatility per second significantly increases likelihood of the PLDL regime occurring.

We also simulate 100 samples using a sample size of 50,000 for each regime of opening hours and trading hours to calculate the related moments of expected trade durations. These simulated moments are generated using coefficient estimates in Panel B of Table 3 and Panel C of Table 4, as well as empirical distributions\textsuperscript{15} of trade durations, price changes, and residuals. Table 7 lists the simulated moments of DG and DL regimes on the market opening. Generally, the DG regime has more positive skewness and highly excess kurtosis than the DL regime. Thus, the trade durations in the DG regime tend to exhibit a large proportion of extremely longer trade durations. Meanwhile, unlike the DL regime, the DG regime does not have symmetric trade durations. Therefore, during opening hours, past longer trade durations are more likely to lead to extremely long trade durations.

Table 8 shows that the PLDG and PLDL regimes have more positive skewness and higher excess kurtosis than the PGDG and PGDL regimes during trading hours. The first moments of the PLDG and PLDL regimes are slightly smaller than those of the PGDG and PGDL regimes. Combined with the higher volatility per second in the PLDG and PLDL regimes, the shorter trade durations are likely to be associated with higher volatility per second, particularly in the downward market. However, trade durations in downward price regimes are also found to be more asymmetric than in upward price regimes and are more likely to have markedly longer trade durations.

### 4.4 Impulse Responses and Conditional Trading Intensity

The generalized impulse response functions (GIRF) developed by Koop et al. (1996) are calculated for different regimes during opening and trading hours. The advantage of GIRF compared to the traditional impulse response function is that

\textsuperscript{15} Empirical distributions can avoid the need to assume specific distributions of input variables and identify the empirical properties of examined data. Empirical distributions are often adopted in the bootstrap and empirical likelihood methods.
Table 7. Simulated Moments for the Opening Hours and Associated Two Regimes

<table>
<thead>
<tr>
<th>Regime</th>
<th>1st Moment</th>
<th>2nd Moment</th>
<th>3rd Moment</th>
<th>4th Moment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Whole Opening Hours</td>
<td>0.3702</td>
<td>0.2386</td>
<td>1.6992</td>
<td>38.5293</td>
</tr>
<tr>
<td></td>
<td>(0.0069)</td>
<td>(0.0252)</td>
<td>(0.7196)</td>
<td>(22.9789)</td>
</tr>
<tr>
<td>DG</td>
<td>0.3920</td>
<td>0.3509</td>
<td>4.1881</td>
<td>105.3145</td>
</tr>
<tr>
<td></td>
<td>(0.0058)</td>
<td>(0.0410)</td>
<td>(1.2295)</td>
<td>(39.3170)</td>
</tr>
<tr>
<td>DL</td>
<td>0.3602</td>
<td>0.1759</td>
<td>0.3047</td>
<td>1.0541</td>
</tr>
<tr>
<td></td>
<td>(0.0060)</td>
<td>(0.0065)</td>
<td>(0.0332)</td>
<td>(0.3889)</td>
</tr>
</tbody>
</table>

a. The simulated moments are based on 100 samples with a sample size of 50,000 for each regime using parameter estimates of Panel B in Table 3. The standard errors are in parentheses.
b. (1) DG regime in which the lagged trade duration change equals or exceeds the mean of trade durations, and (2) the DL regime in which the lagged trade duration change is less than the mean of trade durations.

Table 8. Simulated Moments for the Trading Hours and Associated Four Regimes

<table>
<thead>
<tr>
<th>Regime</th>
<th>1st Moment</th>
<th>2nd Moment</th>
<th>3rd Moment</th>
<th>4th Moment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Whole Trading Hours</td>
<td>0.9962</td>
<td>1.8866</td>
<td>17.3653</td>
<td>300.5614</td>
</tr>
<tr>
<td></td>
<td>(0.0135)</td>
<td>(0.1572)</td>
<td>(3.1166)</td>
<td>(71.4762)</td>
</tr>
<tr>
<td>PGDG</td>
<td>0.9864</td>
<td>0.9076</td>
<td>2.6270</td>
<td>16.5824</td>
</tr>
<tr>
<td></td>
<td>(0.0049)</td>
<td>(0.0187)</td>
<td>(0.2468)</td>
<td>(6.7620)</td>
</tr>
<tr>
<td>PLDG</td>
<td>0.9530</td>
<td>1.5379</td>
<td>13.4570</td>
<td>231.6475</td>
</tr>
<tr>
<td></td>
<td>(0.0127)</td>
<td>(0.1193)</td>
<td>(2.1971)</td>
<td>(49.7260)</td>
</tr>
<tr>
<td>PGDL</td>
<td>0.9830</td>
<td>0.9327</td>
<td>3.0986</td>
<td>26.0487</td>
</tr>
<tr>
<td></td>
<td>(0.0049)</td>
<td>(0.0246)</td>
<td>(0.4094)</td>
<td>(9.6909)</td>
</tr>
<tr>
<td>PLDL</td>
<td>0.9522</td>
<td>1.5120</td>
<td>12.3738</td>
<td>204.0554</td>
</tr>
<tr>
<td></td>
<td>(0.0114)</td>
<td>(0.1140)</td>
<td>(2.0836)</td>
<td>(46.3125)</td>
</tr>
</tbody>
</table>

a. The simulated moments are based on 100 samples with a sample size of 50,000 for each regime using parameter estimates of Panel C in Table 4. The standard errors are in parentheses.
b. (1) PGDG regime in which the lagged price change equals or exceeds zero and the lagged trade duration equals or exceeds the mean of trade durations, (2) the PLDG regime in which the lagged price change is less than zero and the lagged trade duration equals or exceeds the mean of trade durations, (3) the PGDL regime in which the lagged price change equals or exceeds zero and the lagged trade duration is less than the mean of trade durations, and (4) the PLDL regime in which the lagged price change is less than zero and the lagged trade duration is less than the mean of trade durations.

The GIRF considers three factors in calculating the impulse response function of a shock $\varepsilon_i$ on the future duration $x_{i+1}:$ (1) history of the data-generating process until transaction $i,$ (2) size of the shock for transaction $i,$ and (3) shocks between
transactions $i$ and $i + n$ (Koop et al. (1996) and van Dijk and Franses (1999)). Consequently, the GIRF for the current shock $\epsilon_i = \eta_i$ and history $\Omega_{i-1} = \{\delta_{i-d1}, x_{i-d2}\} = w_{i-1}$ is as follows:

$$GIRF(\eta_i, w_{i-1}, n) = E(x_{i+n} \mid \epsilon_i = \eta_i, \Omega_{i-1} = w_{i-1}) - E(x_{i+n} \mid \Omega_{i-1} = w_{i-1})$$

(4.1)

for $n = 1, 2, \ldots, 40$. Restated, the GIRF is the difference between the expected value of $x_{i+n}$, conditional on the current shock and history, and the expected value of $x_{i+n}$, conditional only on the history. The simulation procedure of Koop et al. (1996) is performed by constructing 100 samples with a sample size of 1,000 for each shock size of ±10%, ±50%, and ±90%.

Figures 1 and 2 show generalized impulse responses for two regimes on market opening for the next 40 transactions following the shock to the trade duration while Figs. 5 through 8 illustrate generalized impulse responses for four regimes during trading hours for the next 40 transactions following the shock to the trade duration. Shocks to trade durations on the market opening are found to be more persistent than shocks to trade durations during the trading hours. Consequently, the overnight information coupled with shocks to trade durations leads to persistent expected trade durations on the market opening. Meanwhile, shocks to trade durations of the PGDG and PGDL of upward price regimes are less persistent than those for the PLDG and PLDL of downward price regimes. This indicates that the shocks to trade durations are processed faster in the upward price regime than in the downward price regime. Two factors contribute to the persistence behavior of shocks during the downward market: short sale constraints and higher volatility. Short sale constraints could slow information processing by traders during the downward market. Higher volatility during the downward market reduces the attractiveness of stocks, and infrequent trading slows information revelation. Notably, negative shocks generate slightly larger reductions in trade durations in the upward market but not in the downward market.

The conditional trading intensity, or hazard function, indicates the transaction arrival intensity given past price changes and trade durations. Figures 3 and 4 list the conditional trading intensity for two regimes on the market opening, while Figs. 9 to 12 illustrate the conditional trading intensity for the four regimes during trading hours. The conditional trading intensity is lower for smaller trade
Figure 1. Generalized Impulse Responses of the DG Regime

Figure 2. Generalized Impulse Responses of the DL Regime

Figure 3. Conditional Intensity Function of the DG Regime
Figure 4. Conditional Intensity Function of the DL Regime

Figure 5. Generalized Impulse Responses of the PGDG Regime

Figure 6. Generalized Impulse Responses of the PLDG Regime
Figure 7. Generalized Impulse Responses of the PGDL Regime

Figure 8. Generalized Impulse Responses of the PLDL Regime

Figure 9. Conditional Intensity Function of the PGDG Regime
Figure 10. Conditional Intensity Function of the PLDG Regime

Figure 11. Conditional Intensity Function of the PGDL Regime

Figure 12. Conditional Intensity Function of the PLDL Regime
durations while generally being higher around the middle range of trade durations. This indicates that transactions tend to occur when past transactions were neither too long nor too short, consistent with the findings of Zhang et al. (2001).

5 Conclusions

This study proposed a smooth transition autoregressive conditional duration model of price and duration (STACD-PD) which considers past price changes and trade durations. The STACD-PD model expresses a nonlinear type of the autoregressive conditional duration (ACD) model with a regime-switching feature. The regime switches following smooth transition functions in which the past price changes and past durations are state variables. Accordingly, the STACD-PD model is able to capture the effects of past price changes and past trade durations on the expected trade durations. Meanwhile, the specification tests of the STACD-PD model are also constructed for testing linearity of the expected trade duration process against the smooth transition nonlinearity.

The STACD-PD model was applied to intraday transaction data for the stock of IBM. Empirical results demonstrate nonlinearity in expected trade durations. Notably, past trade durations influence the structural change behavior of expected trade durations on the market opening while past price changes do not. Additionally, expected trade durations become longer and spreads increase in the longer past trade duration regime. Regarding trading hours, the structural change behavior of expected trade durations is affected by both past price changes and past trade durations. The expected trade durations are more persistent during the upward market but not in the downward market. Meanwhile, the expected trade duration increases in situations involving decreasing prices and frequent trades. Regarding the shock effect to the trade durations, shocks to the trade durations on the market opening are more persistent than shocks to trade durations during trading hours. Furthermore, shocks to trade duration fade more quickly in the upward market than in the downward market. Consequently, the STACD-PD model is able to provide a better understanding of the trade duration behavior.
References


Davies, R. B. (1987): “Hypothesis testing when a nuisance parameter is present only under the alternative,” Biometrika, 74, 33–43.


