$B \to \eta(0)^{(I)}(L^{-} \bar{\nu}_{L}, L^{+} L^{-}, K, K^{*})$ decays in the quark-flavor mixing scheme

A. G. Akersoyd,1,2,* Chuan-Hung Chen,1,2,† and Chao-Qiang Geng3,4,‡

1Department of Physics, National Cheng-Kung University, Tainan 701, Taiwan
2National Center for Theoretical Sciences, Taiwan
3Department of Physics, National Tsing-Hua University, Hsinchu 300, Taiwan
4Theory Group, TRIUMF, 4004 Wesbrook Mall, Vancouver, BC V6T 2A3, Canada

(Received 8 January 2007; published 1 March 2007)

In the quark-flavor mixing scheme, $\eta$ and $\eta'$ are linear combinations of flavor states $\eta_{q} = (u \bar{u} + d \bar{d})/\sqrt{2}$ and $\eta_{s} = s \bar{s}$ with the masses of $m_{qq}$ and $m_{ss}$, respectively. Phenomenologically, $m_{ss}$ is strictly fixed to be around 0.69, which is close to $\sqrt{2m_{\pi}^2 - m_{\eta}^{2}}$ by the approximate flavor symmetry, while $m_{qq}$ is found to be 0.18 ± 0.08 GeV. For a large allowed value of $m_{qq}$, we show that the branching ratios (BRs) for $B \to \eta(0)^{0}X$ decays with $X = (\ell^{-} \bar{\nu}_{L}, \ell^{+} \ell^{-})$ are enhanced. We also illustrate that BR($B \to \eta X$) > BR($B \to \eta K^{0}$) in the mechanism without the flavor-singlet contribution. Moreover, we demonstrate that the decay branching ratios for $B \to \eta(0)^{0}K^{*}$ are consistent with the data. In particular, the puzzle of the large BR($B \to \eta K^{*}$) can be solved. In addition, we find that the CP asymmetry for $B^{\pm} \to \eta K^{\pm}$ can be as large as ~30%, which agrees well with the data. However, we cannot accommodate the CP asymmetries of $B \to \eta K^{*}$ in our analysis, which could indicate the existence of some new CP violating sources.

DOI: 10.1103/PhysRevD.75.054003 PACS numbers: 13.20.He

I. INTRODUCTION

The branching ratio (BR) of $B^{0} \to \eta' K^{0}$ was first observed by the CLEO collaboration with $(89 \pm 18 \pm 9) \times 10^{-6}$ [1], which is much larger than $(20 - 40) \times 10^{-6}$ estimated by the factorization ansatz [2]. With more data accumulated, this in comprehensible value becomes a real puzzle now that the measurements from Belle and BABAR depart from the theoretical estimations, where the former has observed BR($B^{+} \to \eta' K^{+}$) = $(1.9 \pm 0.3_{-0.1}^{+0.2}) \times 10^{-6}$ [3], BR($B^{+} \to \eta' K^{0}$) = $(69.2 \pm 2.2 \pm 3.7) \times 10^{-6}$, and BR($B^{0} \to \eta' K^{0}$) = $(58.9 \pm 3.5 \pm 4.3) \times 10^{-6}$ [4], while the latter has measured BR($B^{+} \to \eta' K^{+}$) = $(3.3 \pm 0.6 \pm 0.3) \times 10^{-6}$ [5], BR($B^{+} \to \eta' K^{0}$) = $(68.9 \pm 2.0 \pm 3.2) \times 10^{-6}$, and BR($B^{0} \to \eta' K^{0}$) = $(67.4 \pm 3.3 \pm 3.2) \times 10^{-6}$ [6]. To unravel the mystery, many solutions have been proposed, such as the intrinsic charm in $\eta'$ [7], the gluonium state [8], the spectator hard scattering mechanism [9], and the flavor-singlet component in $\eta'$ [10]. Nevertheless, there are still no conclusive solutions yet.

Recently, the BABAR collaboration [11] has also measured the semileptonic decays of the data as follows:

$$\begin{align*}
\text{BR}(B^{+} \to \eta^{0} \ell^{+} \nu_{\ell}) & = (0.84 \pm 0.27 \pm 0.21) \times 10^{-4} \\
& < 1.4 \times 10^{-4} (90\%C.L.), \\
\text{BR}(B^{+} \to \eta' \ell^{+} \nu_{\ell}) & = (0.33 \pm 0.60 \pm 0.30) \times 10^{-4} \\
& < 1.3 \times 10^{-4} (90\%C.L.).
\end{align*}$$

(1)

Although the significance of the former in Eq. (1) is 2.55σ, the central value is a factor of 2 larger than $0.4 \times 10^{-4}$ calculated by the light-cone sum rules (LCSRs) [12].

II. THE QUARK-FLAVOR MIXING SCHEME

It is known that the physical states $\eta$ and $\eta'$ are composed of the flavor octet $\eta_{8}$ and singlet $\eta_{1}$, in which the flavor wave functions are denoted as $\eta_{8} = (u \bar{u} + d \bar{d} - 2s \bar{s})/\sqrt{6}$ and $\eta_{1} = (u \bar{u} + d \bar{d} + s \bar{s})/\sqrt{3}$, respectively. Because of the $U_{A}(1)$ anomaly, it is understood that the mass of $\eta'$ is much larger than that of $\eta$. To satisfy the current experimental data, usually one needs to introduce two angles to the mixing matrix, defined by $\eta = \cos \theta_{8} \eta_{8} - \sin \theta_{8} \eta_{1}$ and $\eta' = \sin \theta_{8} \eta_{8} + \cos \theta_{8} \eta_{1}$ [2,16], to describe the connection between physical and flavor states. However, it is known that by using the two-angle scheme, we will encounter a divergent problem in some $B$ decays [17], such as $B \to \eta' K$. To illustrate this problem,
we notice that in these decays, the factorized parts are associated with the matrix element \(\langle 0 | i \bar{q} \gamma_5 s | \eta \rangle\). From the equation of motion, one has \(\langle 0 | i \bar{q} \gamma_5 s | \eta \rangle = \langle 0 | 2m, i \bar{q} \gamma_5 s | \eta \rangle = m^2_n f_{\eta} \), leading to \(\langle 0 | i \bar{q} \gamma_5 s | \eta \rangle = m^2_n f_{\eta} / 2m_s\), where \(f_{\eta}(m_{\eta})\) is the decay constant (mass) of \(\eta\). In the chiral limit of \(m_s \to 0\), the matrix element diverges because \(m_n \neq 0\). To explicitly display the chiral limit, it is better to use the quark-flavor scheme, defined by [18,19]

\[
\begin{pmatrix}
\eta \\
\eta_q
\end{pmatrix}
= \begin{pmatrix}
\cos \phi & -\sin \phi \\
\sin \phi & \cos \phi
\end{pmatrix}
\begin{pmatrix}
\eta \\
\eta_q
\end{pmatrix},
\]

where \(\eta_q = (u \bar{d} + d \bar{u})/\sqrt{2}\) and \(\eta_s = s \bar{s}\). From the definition of \(\langle 0 | i \bar{q} \gamma_\mu \gamma_5 q | \eta \rangle(p) = i f_{\eta_q} p_\mu\) \((q' = q, s)\), the masses of \(\eta_q, \eta_s\) can be expressed by

\[
m^2_{\eta_q} = \frac{\sqrt{2}}{f_{\eta_q}} \langle 0 | i \bar{u} \gamma_5 s u + m_d \bar{d} \gamma_5 d | \eta \rangle, \\
m^2_{\eta_s} = \frac{2}{f_{\eta_s}} \langle 0 | i \bar{s} \gamma_5 s | \eta \rangle.
\]

Clearly, in terms of the quark-flavor basis, \(m_{\eta_q}\) and \(m_{\eta_s}\) are zero in the chiral limit. We note that \(m_{\eta_q}\) and \(m_{\eta_s}\) are unknown parameters and their values can be obtained by fitting with the data, such as the masses of \(\eta^{(0)}\) and the decay rates of some relevant \(B\) decays. Note that \(m_{\eta_q, \eta_s}\) are related to \(m_{\eta_q, \eta_s,K}\) by \(m_{\eta_q} = m_{\eta_q}/(m_u + m_d)\), \(m_{\eta_s} = m_{\eta_s}/2m_s\) and \(m_{\eta_K} = m_K^2/(m_s + m_d)\). From the divergences of the axial vector currents

\[
\langle 0 | i \bar{q}'(0) q'_q(z) | \eta_q(p) \rangle = \frac{i}{\sqrt{2N_c}} \int_0^1 dx e^{-ixp_\mu z} \left[ (\tilde{\phi} \gamma_5)_{jk} \phi_{\eta_q} (x) + (\gamma_5)_{jk} m^0_{\eta_q} \phi_{\eta_q} (x) + m^0_{\eta_q} [4\gamma_\mu - 1]_{jk} \phi_{\eta_q} (x) \right],
\]

where \(q' = u, d\) and \(s, q' = q\) and \(s\) denote the twist-2 and twist-3 wave functions of the \(\eta_q\) state, respectively, \(x\) is the momentum fraction, \(m_{\eta_q}^2\) stands for the chiral symmetry breaking parameter, and \(n_s = (1, 0, 0)\) and \(n_d = (0, 1, 0)\) are defined in the light-cone coordinates. On substituting Eq. (7) into Eq. (3), we obtain \(m_{\eta_q}^0 = m_{\eta_q}/(m_u + m_d)\) and \(m_{\eta_s}^0 = m_{\eta_s}/2m_s\). In the next section, it will be clear that the value of \(m_{\eta_q}\) is crucial for the determination of the \(B \to \eta^{(0)} X\) transition form factors, which play important roles in the decay branching ratios of \(B \to \eta^{(0)} X\) with \(X = (\ell^+ \ell^-, K)\).

### III. DECAY AMPATUTES AND FORM FACTORS

We first study the semileptonic decays of \(B^+ \to \eta^{(0)} \ell^+ \nu_\ell\) and \(B^0 \to \eta^{(0)} \ell^+ \ell^-\) by writing the effective Hamiltonians at quark level in the SM as

\[
\mathcal{H}_{ll} = G_F V_{ub} \frac{\bar{u} \gamma_\mu (1 - \gamma_5) b \ell \gamma_\mu (1 - \gamma_5) \nu_\ell}{\sqrt{2}},
\]

\[
\mathcal{H}_{ll} = \frac{G_F V_{ub}}{\sqrt{2}} \bar{u} \gamma_\mu (1 - \gamma_5) b \ell \gamma_\mu (1 - \gamma_5) \nu_\ell.
\]
\[ C_0^{(\ell)}(\mu) = C_0(\mu) + (3C_1(\mu) + C_2(\mu)) \left( h(z, s) - \frac{3}{\alpha_{em}} \sum_{\nu, \nu'} k_{\nu} \frac{\pi \Gamma(V \to \ell^+ \ell^-) M_V}{M_V^2 - q^2 - iM_{\gamma V} \Gamma_V} \right), \]

\[ h(z, s) = \frac{8}{9} \ln \frac{m_b}{\mu} - \frac{8}{9} \ln z + \frac{8}{27} + \frac{4}{9} x \left( 2 + x \right) \left( 1 - x \right)^{1/2} \left[ \ln \left( \frac{1 - x + 1}{\sqrt{1 - x - 1}} \right) - i \pi, \right. \]

\[ \left. 2 \arctan \frac{1}{\sqrt{x - 1}}, \right. \] for \( x = 4z^2/s < 1, \]

\[ \left. 2 \arctan \frac{1}{\sqrt{x - 1}}, \right. \] for \( x = 4z^2/s > 1, \]

where \( h(z, s) \) describes the one-loop matrix elements of operators \( O_1 = \bar{s}_\alpha \gamma^\mu P_\lambda \bar{b}_\mu \bar{\nu}_\gamma \gamma_\mu P_L c_\alpha \) and \( O_2 = \bar{s}_\alpha \gamma^\mu P_\lambda b c_\gamma \gamma_\mu P_L c \) \[ ^{22} \] with \( z = m_c/m_b \) and \( s = q^2/m_b^2, M_V (\Gamma_V) \) are the masses (widths) of intermediate states. The hadronic matrix elements for the \( B \to P \) transition are parametrized as

\[ \langle P(p_P) | \bar{q} \gamma^\mu b | \bar{B}(p_B) \rangle = f_+^P(q^2) \left( \frac{P \cdot q}{q^2} - \frac{q \cdot q}{q^2} \right) + f_0^P(q^2) \frac{P \cdot q}{q^2} q_{\mu}, \]

\[ \langle P(p_P) | \bar{q} i \sigma_{\mu \nu} q \nu | \bar{B}(p_B) \rangle = \frac{f_+^P(q^2)}{m_B + m_P} \left[ P \cdot q q_{\mu} - q^2 P_{\mu} \right], \]

with \( P \) representing the pseudoscalar, \( P_{\mu} = (p_B + p_P)_\mu, q_{\mu} = (p_B - p_P)_\mu \) and \( f_+^P(q^2) \) are form factors. Consequently, the transition amplitudes associated with the interactions in Eqs. (8) and (9) can be expressed as

\[ \mathcal{M}_I = \frac{\sqrt{2} G_F V_{tb} f_+^P(q^2) \ell \tilde{P} \ell}{\pi}, \]

\[ \mathcal{M}_{II} = \frac{G_F \alpha_{em} \ell}{\sqrt{2} \pi} \left[ m_{\gamma_1} E \ell \ell + m_{\gamma_1} E \ell \ell \gamma_5 \ell \right] \]

for \( \bar{B} \to P \ell^+ \ell^- \) and \( \bar{B} \to P \ell^+ \ell^-, \) respectively, where

\[ m_{\gamma_1} = \frac{C_9^{(\ell)} f_+^P(q^2)}{2 m_b} + \frac{2 m_b}{m_B + m_P} C_7 f_0^P(q^2), \]

\[ m_{\gamma_2} = C_1^{(\ell)} f_+^P(q^2). \]

The differential decay rates for \( B^- \to P \ell^+ \ell^- \) and \( \bar{B}_d \to P \ell^+ \ell^- \) as functions of \( q^2 \) are given by \[ ^{12} \]

\[ \frac{d \Gamma_I}{dq^2} = \frac{G_F^2 V_{ub}^2 m_B^3}{3 \times 2^6 \pi^2} \left( 1 - s + m_{\gamma_1}^2 \right) \sqrt{1 - s + m_{\gamma_1}^2 - 4 m_{\tilde{P}}^2(f_+^P(q^2) \hat{P}_\ell)^2}, \]

\[ \frac{d \Gamma_{II}}{dq^2} = \frac{G_F^2 \alpha_{em} m_B^3}{3 \times 2^9 \pi^2} \left( 1 - s + m_{\gamma_2}^2 \right) \sqrt{1 - s + m_{\gamma_2}^2 - 4 m_{\tilde{P}}^2} \times \hat{P}_\ell \left( |m_{\gamma_1}|^2 + |m_{\gamma_2}|^2 \right). \]

The form factors for \( B \to P \) can be formulated as \[ ^{23} \]

\[ f_+^P(q^2) = f_0^P(q^2) + f_2^P(q^2), \]

\[ f_0^P(q^2) = f_0^P(q^2) \left( 1 + \frac{q^2}{m_B^2} \right) + f_2^P(q^2) \left( 1 - \frac{q^2}{m_B^2} \right), \]

where

\[ f_2^P(q^2) = 8 \pi C_F m_B^2 r_P \int_0^1 [dx] \int_0^1 b_1 d b_1 d b_2 d b_2 \phi_2(x_1, b_1) [\phi_2(x_2) - \phi_2(x_2)] E(t^{(1)}) \hat{h}(x_1, x_2, b_1, b_2), \]

\[ f_2^P(q^2) = 8 \pi C_F m_B^2 \int_0^1 [dx] \int_0^1 b_1 d b_1 d b_2 d b_2 \phi_2(x_1, b_1) \left[ (1 + x_2 \xi) \phi_2(x_2) + 2 r_P \left( \frac{1}{\xi} - x_2 \phi_2(x_2) \right) \right] \]

\[ \times E(t^{(2)}) h(x_1, x_2, b_1, b_2) + 2 r_P \phi_2(x_2) E(t^{(2)}) h(x_1, x_2, b_1, b_2). \]
\[ f^P_T(q^2) = 8\pi C_F m_B^2 (1 + m_P/m_B) \int_0^1 dx \int_0^\infty b_1 b_2 b_3 db_3 \phi_B(x_1, b_1) \left[ \phi_P(x_2) - r_P x_2 \phi_P^0(x_2) + r_P \left( \frac{2}{\xi} + x_2 \right) \phi_P^0(x_2) \right] \times E(t^{(1)}) h(x_1, x_2, b_1, b_2) + 2 r_P \phi_P^0(x_2) E(t^{(2)}) h(x_2, x_1, b_2, b_1), \]

(20)

with \( C_F = 4/3, \xi = 1 - q^2/m_B^2, \) and \( r_P = m_P^2/m_B. \) From Eq. (18), we find that \( f_+(0) = f_0(0). \) The evolution factor is given by \( E(t) = \alpha_s(t) \exp(-S_B(t) - S_P(t)) \) where the Sudakov exponents \( S_B(t) \) can be found in Ref. [26]. The hard function \( h \) is written as

\[ h(x_1, x_2, b_1, b_2) = S_i(x_2) K_0(\sqrt{x_1 x_2} \xi m_B b_1) \left[ \theta(b_1 - b_2) - \theta(b_2 - b_1) K_0(\sqrt{x_1 x_2} \xi m_B b_1) I_0(\sqrt{x_1 x_2} \xi m_B b_2) \right], \]

(21)

where the threshold resummation effect is described by \( S_i(x) = 2^{1 + z} \Gamma(\frac{3}{2} + c)[x(1 - x)]^{1/2} \sqrt{\pi} \Gamma(1 + c) \) with \( c = 0.3 \) [25]. The hard scales \( t^{(1,2)} \) are chosen to be [27]

\[ t^{(1)} = \max(\sqrt{m_B^2 \xi x_1}, 1/b_1, 1/b_2, \tilde{\Lambda}), \]
\[ t^{(2)} = \max(\sqrt{m_B^2 \xi x_1}, 1/b_1, 1/b_2, \tilde{\Lambda}), \]

where \( \tilde{\Lambda} \) is used to exclude the effects from nonperturbative contributions. To get the BRs for the three-body semileptonic decays, besides the values of the form factors at \( q^2 = 0, \) we also need to know their \( q^2 \) dependences. To obtain them, we adopt the fitting results calculated by the light-cone sum rules (LCSR) [28], given by

\[ f^{\eta_T}_+(q^2) = \frac{f^{\eta_T}_+(0)}{(1 - q^2/m_B^2)(1 - \alpha_T q^2/m_B^2)} \]

(22)

with \( \alpha_T = 0.52(0.84) \) and \( m_{\eta_c} = 5.32 \) GeV. In terms of the quark-flavor mixing scheme, we will calculate the \( B \to \eta_{q,s} \) form factors, which are related to those of \( B \to \eta^{(i)} \) by

\[ f^{\eta_T}_+(q^2) = \frac{\cos \phi}{\sqrt{2}} f^{\eta_{q,s}}_+(q^2), \]
\[ f^{\eta_T}_+(q^2) = \frac{\sin \phi}{\sqrt{2}} f^{\eta_{q,s}}_+(q^2). \]

(23)

For the nonleptonic decays of \( B \to \eta^{(i)} K, \) we will assume the color transparency [29], i.e., no rescattering effects in \( B \) decays. The effective interaction for the \( b \to s \) transition at the quark level is given by [22]

\[ 0 \quad 0 \]
\[ \Gamma^\mu \]
\[ b \quad q' \]

FIG. 1 (color online). Flavor diagrams for the \( B \to P \) transition with \( \Gamma^\mu = (\gamma^\mu, i\sigma^{\mu\nu} q_\nu). \)
we have to know not only the relevant effective weak interactions but also all possible topologies for the specific process. In Fig. 2, we display the flavor diagrams for $B_d \to \eta_{q,s} K$ decays, in which (2a)–(2c), (2d) and (2e), and (2f) illustrate penguin emission, penguin annihilation, and tree emission topologies, respectively. Since the $b$-quark is dictated by the weak charged current, its chirality is always left handed. However, the chiralities for $q\bar{q}$ pairs, produced by gluon, $Z$-boson, and photon penguins, could be both left and right handed, resulting in processes containing both $V-A$ and $V+A$ currents. In Fig. 2, we have explicitly labeled the associated type of currents except the diagram 2(f) which is from the tree and only has the left-handed interaction. Note that although we use the states $\eta_{q,s}$ as our basis, the physical states can be easily obtained by using Eq. (2). For the charged $B$ decays, besides the flavor diagrams displayed in Fig. 2, three more diagrams arising from tree emission and annihilation topologies need to be included as shown in Fig. 3. From Figs. 2 and 3, the decay amplitudes for $B^{0,+} \to \eta_{q}K^{*(0),+}$ and $B \to \eta_{q}K^{*(0),+}$ are given by

$$A_{q}^{0} = V_{t}(F_{Pa}^{0} + N_{Pa}^{0} + F_{Pe}^{0} + N_{Pe}^{0} + F_{pd}^{0} + N_{pd}^{0})$$

$$- V_{u}(F_{Tf}^{0} + N_{Tf}^{0}),$$

$$A_{q}^{0} = V_{t}(F_{P(b+c)}^{0} + N_{P(b+c)}^{0} + F_{Pe}^{0} + N_{Pe}^{0}),$$

(26)

and

FIG. 2. Flavor diagrams for $B_d \to \eta_{q,s} K$ decays: (a)–(f) stand for the penguin emission contributions while (f) is the tree contribution, where $V \pm A$ denote the left-handed and right-handed currents, respectively.

FIG. 3. Flavor diagrams arising from tree emission and annihilation for charged $B$ decays.

The decay BRs and CP asymmetries (CPAs) are given by

$$\frac{\text{BR}(B^{0,+} \to \eta_{q}K^{*(0),+})}{16\pi} = \frac{G_{F}^{2}|\bar{\rho}|m_{B}^{2}T_{B}^{0,+}}{A(B^{0,+} \to \eta_{q}K^{*(0),+})}$$

$$A_{CP}(B \to \eta_{q}K^{*}) = \frac{-\text{BR}(B \to \eta_{q}K^{*})}{\text{BR}(B \to \eta_{q}K^{*})} - \text{BR}(B \to \eta_{q}K^{*})$$

(29)

(30)

which can be evaluated in terms of Eqs. (26)–(28), where $|\bar{\rho}| = \sqrt{E_{K}^{2} - m_{K}^{2}}$ and $E_{K} = (m_{B}^{2} - m_{\eta_{q}}^{2} + m_{K}^{2})/2m_{B}$.

IV. NUMERICAL RESULTS AND DISCUSSIONS

In the PQCD approach, if we regard the meson wave functions as known objects, the remaining unknown theoretical quantities are the chiral symmetry breaking parameters of states $\eta_{q,s}$ and $K$, denoted by $m_{0_{q,s},q,s} K$, and the meson decay constants $f_{B,q_{q,s},K}$. It is known that $f_{K}$ has been determined quite precisely to be around 0.16 GeV by experiment, while the lattice QCD calculations give $f_{K} = 0.216 \pm 0.022$ GeV [31], which is consistent with the extracted value from the decay $B \to \tau_{\nu}$ measured by Belle [32]. By low-energy experiments, the decay constants of $\eta_{q,s}$ are found to be $f_{\eta_{q}} = (1.07 \pm 0.02)f_{\pi}$ and $f_{\eta_{s}} = (1.34 \pm 0.06)f_{\pi} [19]$, respectively. Basically, the undetermined parameters in our considerations are the parameters $m_{qq}$ and $m_{s}$. To obtain the allowed range for $m_{qq,s}$ in a model-independent way, we adopt the phenomenological approach. The parameters in Eq. (6) are limited.
to be \( \phi = 39.3^\circ \pm 1.0^\circ, \quad \gamma = 0.81 \pm 0.03, \) and \( a^2 = 0.265 \pm 0.010 \) [19]. With these values, the allowed ranges for \( m_{qq} \) and \( m_{ss} \) are presented in Fig. 4. From the figure, we find that \( m_{ss} \) has a narrow allowed window around 0.69 GeV, which can be understood in terms of the flavor symmetry, given by \( m_{ss} = \sqrt{2m_K^2 - m_\pi^2} \) [20]. However, \( m_{qq} \) is relatively broader, given by 0.18 \( \pm 0.08 \) GeV. To do the numerical estimations, we take \( f_B = 0.19 \) GeV, \( f_{\eta_s} = 0.14 \) GeV, and \( \phi = 39.3^\circ \) as the input values. For the nonperturbative wave functions, we use the results derived by the LCSR for the light mesons [28], while for the \( B \) meson wave function, we use

\[
\phi_B(x) = N_B x^2 (1 - x)^2 \exp \left[ -\frac{m_{B}^2 x^2}{2\omega_B} \right] \exp \left[ -\frac{\omega_B b^2}{2} \right]
\]

with \( N_B = 111.2 \) and \( \omega_B = 0.38 \) [26]. Accordingly, we get the \( B \to K \) form factor of \( f_+^B(0) \), defined in Eq. (12), to be 0.36. From Eq. (23), we show the form factors \( f_{\pm,T}^q(0) \) in Table I. From the table, we see clearly that they will be enhanced with increasing \( m_{qq} \). In addition, it is easy to understand that the behavior \( f_{\pm,T}^\eta(0) > f_{\pm,T}^q(0) \) is always satisfied as seen from Eq. (23) due to \( \cos \phi > \sin \phi \) with \( \phi \sim 39.3^\circ \). This property is different from that in the FSM, given by [10]

\[
f_i^q(0) = \frac{\cos \phi}{\sqrt{2}} \frac{f_q}{f_\pi} f_+^q(0) + \frac{1}{\sqrt{3}} \left( \sqrt{2} \cos \phi \frac{f_q}{f_\pi} - \sin \phi \frac{f_s}{f_\pi} \right) f_i^{\text{sing}}(0),
\]

\[
f_i^\eta(0) = \frac{\sin \phi}{\sqrt{2}} \frac{f_q}{f_\pi} f_+^q(0) + \frac{1}{\sqrt{3}} \left( \sqrt{2} \sin \phi \frac{f_q}{f_\pi} + \cos \phi \frac{f_s}{f_\pi} \right) f_i^{\text{sing}}(0),
\]

where \( f_i^{\text{sing}}(0)(i = +, T) \) correspond to the new form factors due to the flavor-singlet state. Based on \( f_+^T(0) = f_+^q(0) \approx 0.26 \) calculated by the LCSRs [28], we present the numerical results of Eq. (32) in Table II. From the table, we see that \( f_{\pm,T}^q(0) < f_{\pm,T}^\eta(0) \) in the FSM. Furthermore, by using \( |V_{ub}| = 3.5 \times 10^{-3}, \quad |V_{us}| = 8.1 \times 10^{-3} \) [33], Eqs. (16), (17), and (22), and the values in Tables I and II, we show the semileptonic decay BRs in Table III. From the table, we find that the results in both approaches could be consistent with the data of \( B^- \to \eta(0) \ell \nu_\ell \). On the other hand, in our approach, we always predict \( \text{BR}(B^- \to \eta(0) \ell^+ \nu_\ell) > \text{BR}(B^- \to \eta(0) \ell^+ \nu_\ell) \), whereas the inequality is reversed in the FSM. A similar conclusion can also be drawn for the processes of \( B_d \to \eta(0) \ell^+ \ell^- \). We note that the BRs are insensitive to the parametrizations displayed in Eq. (22) [12].

We now give our numerical analysis for the nonleptonic decays \( B \to \eta(0)K^{(*)} \). By using the PQCD approach, the values of factorized and nonfactorized contributions for the \( B \) decays are shown in Table IV. Based on these values and \( V_{ts} = -0.041 \) and \( V_{ub} = 4.6 \times 10^{-3} e^{-i\phi_b} \) with \( \phi_b = 72^\circ \), the predictions for \( \text{BR}(B \to \eta(0)K^{(*)}) \) and \( A_{CP}(B \to \eta(0)K^{(*)}) \) are given in Tables V and VI, respectively. Our results can be summarized as follows:

(i) From Table V, we see clearly that with \( m_{qq} = 0.22 \) GeV, the BRs for \( B \to \eta(0)K^{(*)} \) are consistent with the world average (WA) data. It is interesting to note that by increasing \( m_{qq} \), \( \text{BR}(B \to \eta(0)K) \) tend to be small (large), while \( \text{BR}(B \to \eta(0)K^*) \) tend to be large (small), favored by the experiments.

(ii) As seen from Table V, with the same value of \( m_{qq} \), \( \text{BR}(B \to \eta K) < O(10^{-3}) \text{BR}(B \to \eta' K) \), while \( \text{BR}(B \to \eta K^*) > \text{BR}(B \to \eta' K^*) \). The phenomena could be ascribed to the signs in the amplitudes of \( B \to (\eta_s, \eta_s')K^{(*)} \) by comparing Eqs. (26)–(28) with the specific values of \( F_{Pa}^{0,+} \) and \( F_{P(b+c)}^{0,+} \) in Table IV.

![FIG. 4. The allowed ranges for \( m_{qq} \) and \( m_{ss} \).](image-url)

| TABLE I. \( f_i^q(0) \) and \( f_i^{\eta}(0) \) with three allowed values of \( m_{qq} \). |
|----------------|----------------|----------------|----------------|----------------|
| \( m_{qq} \) (GeV) | \( f_i^q(0) \) | \( f_i^{\eta}(0) \) | \( f_i^{\eta}(0) \) | \( f_i^{\eta}(0) \) |
| 0.14 | 0.14 | 0.14 | 0.12 | 0.11 |
| 0.18 | 0.21 | 0.20 | 0.18 | 0.17 |
| 0.22 | 0.29 | 0.29 | 0.24 | 0.24 |

![TABLE II. \( f_i^{\text{sing}}(0) \) with various values of \( f_i^{\text{sing}}(0) \) in the FSM.](image-url)

<table>
<thead>
<tr>
<th>( f_i^{\text{sing}}(0) )</th>
<th>( f_i^{\text{sing}}(0) )</th>
<th>( f_i^{\text{sing}}(0) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.15</td>
<td>0.13</td>
</tr>
<tr>
<td>0.1</td>
<td>0.20</td>
<td>0.25</td>
</tr>
<tr>
<td>0.2</td>
<td>0.25</td>
<td>0.38</td>
</tr>
</tbody>
</table>
TABLE III. BRs of $B^- \to \eta^0 \ell^+ \ell^-$ and $\bar{B}_d \to \eta^0 \ell^+ \ell^-$ (in units of $10^{-4}$) with $m_{qq} = 0.14, 0.18, \text{and } 0.22 \text{ GeV}$ in our mechanism and $f_{\eta^0}^{\text{fin}}(0) = 0.0, 0.1, \text{and } 0.2$ in the FSM. $\phi = 39.3^\circ$.

<table>
<thead>
<tr>
<th>$m_{qq}$ (GeV)</th>
<th>$B^- \to \eta^0 \ell^+ \ell^-$</th>
<th>$\bar{B}_d \to \eta^0 \ell^+ \ell^-$</th>
<th>$B^- \to \eta^0 \ell^+ \ell^-$</th>
<th>$\bar{B}_d \to \eta^0 \ell^+ \ell^-$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.14</td>
<td>0.30</td>
<td>0.15</td>
<td>0.02</td>
<td>0.03</td>
</tr>
<tr>
<td>0.18</td>
<td>0.67</td>
<td>0.35</td>
<td>0.04</td>
<td>0.02</td>
</tr>
<tr>
<td>0.22</td>
<td>1.27</td>
<td>0.62</td>
<td>0.07</td>
<td>0.04</td>
</tr>
<tr>
<td>$f_{\eta^0}^{\text{fin}}(0)$</td>
<td>$B^- \to \eta^0 \ell^+ \ell^-$</td>
<td>$\bar{B}_d \to \eta^0 \ell^+ \ell^-$</td>
<td>$B^- \to \eta^0 \ell^+ \ell^-$</td>
<td>$\bar{B}_d \to \eta^0 \ell^+ \ell^-$</td>
</tr>
<tr>
<td>0.0</td>
<td>0.38</td>
<td>0.18</td>
<td>0.05</td>
<td>0.03</td>
</tr>
<tr>
<td>0.1</td>
<td>0.47</td>
<td>0.64</td>
<td>0.07</td>
<td>0.10</td>
</tr>
<tr>
<td>0.2</td>
<td>0.58</td>
<td>1.39</td>
<td>0.08</td>
<td>0.21</td>
</tr>
<tr>
<td>Exp.</td>
<td>$0.84 \pm 0.27 \pm 0.21(1.4)$</td>
<td>$0.33 \pm 0.60 \pm 0.30(1.3)$</td>
<td>—</td>
<td>—</td>
</tr>
</tbody>
</table>

TABLE IV. Factorizable and nonfactorizable parts for the decays $B^- \to \eta_{c_0}^0 K^\mp$ with $m_{qq} = 0.22 \text{ GeV}$, where the values in the square brackets are for $B^- \to \eta_{c_0}^0 K^-$.  

<table>
<thead>
<tr>
<th>$F_{P_d}^{0}$</th>
<th>$N_{P_d}^{0}$</th>
<th>$F_{P(b+c)}^{0}$</th>
<th>$N_{P(b+c)}^{0}$</th>
<th>$F_{P_d}^{0}$</th>
<th>$N_{P_d}^{0}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-1.10$</td>
<td>$6.36 \pm 2.06$</td>
<td>$-0.55$</td>
<td>$-0.33 + i1.22$</td>
<td>$0.26$</td>
<td>$-7.17 + i3.39$</td>
</tr>
<tr>
<td>$[-0.42]$</td>
<td>$[3.89 \pm 2.37]$</td>
<td>$[0.45]$</td>
<td>$[-0.89 + i1.74]$</td>
<td>$[0.09]$</td>
<td>$[-7.88 + i2.56]$</td>
</tr>
<tr>
<td>$F_{P_d}^{3}$</td>
<td>$N_{P_d}^{3}$</td>
<td>$F_{P(b+c)}^{3}$</td>
<td>$N_{P(b+c)}^{3}$</td>
<td>$F_{P_d}^{3}$</td>
<td>$N_{P_d}^{3}$</td>
</tr>
<tr>
<td>$-0.61 + i2.43$</td>
<td>$-5.77 \pm 9.62$</td>
<td>$-0.44 + i1.25$</td>
<td>$-0.51 - i4.62$</td>
<td>$-0.61$</td>
<td>$3.61 \pm 1.57$</td>
</tr>
<tr>
<td>$[-0.19 + i2.37]$</td>
<td>$[-5.05 \pm 1.36]$</td>
<td>$[0.30 - i1.86]$</td>
<td>$[-5.02 - i9.06]$</td>
<td>$[-0.41]$</td>
<td>$[4.00 - i1.29]$</td>
</tr>
<tr>
<td>$F_{P_d}^{3}$</td>
<td>$N_{P_d}^{3}$</td>
<td>$F_{P(b+c)}^{3}$</td>
<td>$N_{P(b+c)}^{3}$</td>
<td>$F_{P_d}^{3}$</td>
<td>$N_{P_d}^{3}$</td>
</tr>
<tr>
<td>$-1.05$</td>
<td>$3.55 \pm 9.27$</td>
<td>$-0.54$</td>
<td>$-3.56 + i1.71$</td>
<td>$0.21$</td>
<td>$-6.24 + i1.70$</td>
</tr>
<tr>
<td>$[-0.43]$</td>
<td>$[-1.86 \pm i2.61]$</td>
<td>$[0.45]$</td>
<td>$[-0.89 + i1.74]$</td>
<td>$[0.08]$</td>
<td>$[-7.64 + i3.53]$</td>
</tr>
<tr>
<td>$F_{P_d}^{3}$</td>
<td>$N_{P_d}^{3}$</td>
<td>$F_{P(b+c)}^{3}$</td>
<td>$N_{P(b+c)}^{3}$</td>
<td>$F_{P_d}^{3}$</td>
<td>$N_{P_d}^{3}$</td>
</tr>
<tr>
<td>$-0.63 + i2.20$</td>
<td>$-2.86 \pm 9.46$</td>
<td>$-0.50 + i1.60$</td>
<td>$-1.59 - i2.98$</td>
<td>$-0.45$</td>
<td>$3.27 - i0.90$</td>
</tr>
<tr>
<td>$[-0.09 + i2.37]$</td>
<td>$[-2.21 - i0.72]$</td>
<td>$[0.29 - i1.83]$</td>
<td>$[-2.83 - i3.70]$</td>
<td>$[-0.41]$</td>
<td>$[3.85 - i1.74]$</td>
</tr>
<tr>
<td>$F_{T_g}^{3}$</td>
<td>$N_{T_g}^{3}$</td>
<td>$F_{T(b+c)}^{3}$</td>
<td>$N_{T(b+c)}^{3}$</td>
<td>$F_{T_g}^{3}$</td>
<td>$N_{T_g}^{3}$</td>
</tr>
<tr>
<td>$10.03$</td>
<td>$-1.16 + i0.18$</td>
<td>$2.38 + i0.02$</td>
<td>$0.99 + i3.38$</td>
<td>$-1.02 - i0.02$</td>
<td>$0.27 + i1.13$</td>
</tr>
<tr>
<td>$[11.60]$</td>
<td>$[-1.55 + i0.06]$</td>
<td>$[-2.19 - i1.14]$</td>
<td>$[1.09 + i0.70]$</td>
<td>$[1.61 + i0.95]$</td>
<td>$[0.89 + i1.55]$</td>
</tr>
</tbody>
</table>

(iii) From Table VI, we find that for $m_{qq} = 0.22 \text{ GeV}$, $A_{CP}(B_u \to \eta K^+)$ is as large as $-30\%$, which agrees well with the data, whereas the other two sets of $m_{qq}$ lead to positive and small asymmetries. In addition, our prediction for $A_{CP}(B_d \to \eta K^{0*})$ is too small, while that of $A_{CP}(B_u \to \eta K^{*+})$ is too large, in comparison with the data. If future experiments display the current tendencies for these CPAs, such phenomena will become new puzzles.

Finally, we remark that in the quark-flavor scheme, as the errors in the decay constants of $f_q$ and $f_s$ are only 2%
and 4%, respectively, their effects on BRs and CPAs are mild. However, the influence from the mixing angle $\phi$ could be larger. We present the results with the error of $\phi$ in Table VII.

### V. CONCLUSIONS

Because of the current experimental limits on the mixing parameters of the $\eta$ and $\eta'$ mesons, we have studied the phenomenologically allowed ranges for $m_{ss}$ and $m_{qq}$. Explicitly, we have found that $m_{ss}$ is around 0.69 GeV and $m_{qq} = 0.18 \pm 0.08$ GeV. We have shown that the semileptonic decays of $B \to \eta(0)\ell\bar{\nu}_\ell$ are sensitive to $m_{qq}$ and thus they can provide strong constraints on its value. In addition, our mechanism based on the quark-flavor mixing scheme naturally leads to $f_q^\eta(0) < f_q^{\eta'}(0)$ as well as $\text{BR}(B^- \to \eta\ell^-\bar{\nu}_\ell) > \text{BR}(B^- \to \eta'\ell^-\bar{\nu}_\ell)$, in contrast with the reversed inequalities in the FSM due to the flavor-singlet contribution [12,13]. Similar conclusions can also be drawn for the decays $B_d \to \eta(0)\ell^+\ell^-$. It is interesting to note that the future measurements on $\text{BR}(B^- \to \eta(0)\ell\bar{\nu}_\ell)$ and $\text{BR}(B_d \to \eta(0)\ell^+\ell^-)$ can be used to distinguish the two flavor mechanisms. Moreover, we have shown that $\text{BR}(B \to \eta(0)X)$ with $X = (\ell^-\bar{\nu}_\ell, \ell^+\ell^-)$ are enhanced and, in particular, the puzzle of the large $\text{BR}(B \to \eta')$ can be solved with a reasonable large value of $m_{qq}$. We have also demonstrated that $A_{CP}(B^\pm \to \eta K^{\mp})$ can be as large as $-30\%$ and $\text{BR}(B \to \eta(0)K^*)$ are consistent with the current data. Finally, we remark that our results for $A_{CP}(B \to \eta K^*)$ do not agree with the experimental values. According to our analysis, currently, they are the most incomprehensible phenomena. Other mechanisms as well as more precise measurements are needed for a complete description of all the above decays.

### ACKNOWLEDGMENTS

The authors would like to thank Professor Hai-Yang Cheng and Professor Hsiang-Nan Li for useful discussions. This work is supported in part by the National Science Council of R.O.C. under Grant No. NSC-95-2112-M-006-013-MY2 and No. NSC-95-2112-M-007-059-MY3.
[34] E. Barberio et al. (Heavy Flavor Averaging Group), hep-ex/0603003; online update at http://www.slac.stanford.edu/xorg/hfag.