Novel two-loop SUSY effects on CP asymmetry in $B \to \phi K_s$

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Inspired by the exotic measurements on the CP asymmetry in $B \to \phi K_s$, we study a new diagram in supersymmetric models which can make the difference $\sin 2\phi_1^{\psi K_s} - \sin 2\phi_1^{\phi K_s}$ to be 20–50% after satisfying the constraint from $b \to s \gamma$. We also find that the direct CP asymmetry of $b \to s \gamma$ could be $\sim 10\%$ and testable at $B$ factories.

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While enjoying the large CP asymmetry (CPA) in the decay of $B \to J/\Psi K_s$ observed by Belle$^1$ and Babar$^2$ at the precision level, the recent data on $B \to \pi^+\pi^-$ and $B \to \phi K_s$$^3,4$ have stimulated theorists to think more about other possible CP violating phases, beside the Kobayashi-Maskawa (KM)$^5$ phase in the standard model (SM).

It is known that with the Wolfenstein parametrization$^6$, the tree and penguin diagrams have the same CP phase for the inclusive processes of $b \to scc$ and $b \to sss$. Thus, the time-dependent CPA, proportional to $\Gamma(B \to f_{CP}) - \Gamma(B \to f_{\bar{CP}})$ with $f_{CP}$ being the final state and having a definite CP property, arises from the $B - \bar{B}$ oscillation dictated by box diagrams, in which the source of the CP phase is from $V_{td}$ and $e^{-i\phi_S}$. For the channel of $f_{CP} = \phi K_s$, the CPA is related to $\sin 2\phi_1$ and the mixing-induced CP violation. If there is only the KM phase involved in the low-energy, the pure penguin process of $B \to \phi K_s$ has approximately the same value of $\sin 2\phi_1$ as that in the decay of $B \to J/\Psi K_s$, i.e., $\Delta S_{\phi_1} = \sin 2\phi_1(\phi K_s) - \sin 2\phi_1(\phi K_s) \approx 0$.

It is usually believed that new physics could go into low-energy phenomena through loop diagrams, in which new particles appearing in the loops are integrated out and the remaining effective couplings are as functions of their masses and couplings to the conventional particles. Since the transition of $b \to sss$ is a pure quantum loop effect, one can recognize immediately that $B \to \phi K_s$ is a good candidate to probe new physics. Furthermore, although the tree-level contributions in $b \to scc$ are over a factor of 5 larger than those of penguin diagrams in the SM$^6$, the penguin-type diagrams induced by new physics could be enhanced, which will clearly affect the decay of $B \to J/\Psi K_s$, especially on its direct CPA.

To understand the Belle’s result of the 3.5σ difference on $\sin 2\phi_1$ between $J/\Psi K_s$ and $\phi K_s$ modes$^4$, various theoretical models such as those with supersymmetry (SUSY)$^7,8,9,10$ and left-right symmetry$^11$ have been investigated. In addition, the authors of Refs.$^10,11$ have tried to solve the problem of unexpected large branching ratios (BRs) in $B \to \eta' K$ decays. However, we would like to address some problems on these attempts as follows:

(a) Direct CP violation on $B \to J/\Psi K_s$:

We emphasize that Belle and Babar not only measure an accurate mixing-induced CPA, but also indicate no direct CPA in $B \to J/\Psi K_s$, up to the percentage level. Those new SM-like effective four-fermion interactions for $b \to sss$ will inevitably contribute to $b \to scc$. It is also known that there exist large strong phases in the production of charmed mesons (including charmonium states)$^{12,13}$. Therefore, to enhance the BRs of $B \to \eta' K_s$ with large CP violating effects will make the direct CPA in $B \to J/\Psi K_s$ to be over the current experimental limits.

(b) BRs of $B \to \eta K$ and $B \to \eta^{(i)} K^*$:

We note that the problems for the production of $\eta'$ in $B$ decays depend on not only $B \to \eta K$, but also $B \to \eta K$ and $B \to \eta^{(i)} K^*$. From the data at Babar, we have that BR$(B \to \eta K^0) = (2.9 \pm 1.0 \pm 0.2) \times 10^{-6}$, BR$(B \to \eta K^0) = (18.6 \pm 2.3 \pm 1.2) \times 10^{-6}$, and BR$(B \to \eta' K^*) < 7.6 \times 10^{-6}$, by using the perturbative QCD approach$^{10}$, we find that the estimating BRs of $B \to \eta K^0$ and $\eta' K^*$ are over the current experimental values, whereas it is lower for $B \to \eta K^0$.

It is clear that to resolve the problems we need more knowledge on $\eta^{(i)}$ mesons as well as their relevant physics. On the other hand, we may bypass these problems by concentrating on new physics effects which are insensitive to hadronic uncertainties. In this paper, we will introduce a two-loop diagram illustrated in Fig. 1 in the framework of SUSY models, resulting from dipole operators. In contrast with other mechanisms, such as those discussed in Refs.$^{10,12}$ in which the relevant off-diagonal terms of squark-mass matrices directly involve flavor changing neutral current that couples to gluino, our two-loop effect shows how to generate the flavor changing processes naturally in the SUSY models. We will illustrate that the diagram not only contributes a sizable value for the difference of $\sin 2\phi_1$ between $J/\Psi K_s$ and $\phi K_s$ channels, but also satisfies the experimental constraints such as those from the $b \to s \gamma$ decay and the neutron electric dipole moment (NEDM). Since the diagram involves the couplings of the charged Higgs to squarks, we first discuss the relevant couplings in SUSY

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models, given by \cite{20}

\[
\mathcal{L}_{Hff} = -(2\sqrt{2}G_F)^{1/2} \left( \bar{V}_{tb} \tilde{A}_L^b \tilde{t}_L \tilde{b}_R + \bar{V}_{ta} \tilde{A}_R^a \tilde{t}_R \tilde{s}_L \right) H^+ + h.c., \tag{1}
\]

where \( \tilde{A}_L^b = m_b (A_b^b \tan \beta - \mu) \) and \( \tilde{A}_R^a = m_t (A_t \cot \beta - \mu^*) \). Here, the definition of the angle \( \beta \) is followed by \( \tan \beta = v_u/v_d \) with \( v_u \) and \( v_d \) being the vacuum expectation values (VEVs) of Higgs fields \( \Phi^u \) and \( \Phi^d \) responsible for the masses of upper and down type quarks, respectively, and \( \mu \) is the mixing effects of \( \Phi^u,d \). For a large \( \tan \beta \) case, \( \tilde{A}_L^b \) and \( \tilde{A}_R^a \) can be simplified as \( \tilde{A}_L^b \approx m_b A_b^b \tan \beta \) and \( \tilde{A}_R^a \approx -m_t \mu^* \). Note that we have neglected the contribution of \( \tilde{s}_R \) because the corresponding coupling is associated with the strange-quark mass. Moreover, in order to suppress one-loop contributions, we assume that the flavor mixing effects on the down-type squark mass matrix are small.

We remark that both flavor changing and chirality flipping are involved in Fig. 1 in which the charged Higgs is used to change the flavor and the mixing of \( \tilde{t}_L \) and \( \tilde{t}_R \) to govern the chirality flipping, representing by the cross in the figure. Explicitly, as usual, the mixing terms are described by \cite{21}

\[
(\mathbf{m}_u^2)_{LR} = (\mathbf{M}_U^2)_{LR} - \mu \cot \theta m_u,
\]

where \( (\mathbf{M}_U^2)_{LR} \) represent the trilinear soft breaking effects. For simplicity, we have adopted the so-called super-CKM basis, where quarks are in the mass eigenstates so that \( m_u \) is the diagonal upper-type quark mass matrix \cite{21}. To overcome the NEDM constraint, it has been proposed \cite{22} to use hermitian Yukawa and \( A \) matrices. The construction of a hermitian Yukawa matrix can be implemented based on some symmetries, such as the horizontal \( SU(3)_H \) \cite{23} and left-right \cite{24} symmetries. As a result, the CP phases of \( O(1) \) can exist naturally even with the NEDM contributions. Moreover, it implies that the CP asymmetries in hyperon decays could reach the value of \( O(10^{-4}) \) \cite{25}, which is testable in the experiment E871 at Fermilab \cite{26}. However, in the class of models proposed in Ref. \cite{22}, the \( \mu \) parameter is real which is not favored in our following discussions. To avoid this shortcoming, we address the NEDM constraint by imposing the Yukawa and \( A \) matrices to be hermitian and the squark mass of the first generation to be \( \mathcal{O}(10) \) TeV. Hence, the \( \mu \) parameter is regarded as a complex value in our approach. Due to the hermitian property, a special relation is obtained as \( (\delta_{kl}^U)_{LR} = (\delta_{kl}^U)_{RL} \) with \( (\delta_{kl}^U)_{LR} \equiv (\mathbf{M}_U^2)_{kl}^{LR}/m^2 = (V^{U \dagger} A^U v_u V^{U \dagger})_{kl}/m^2 \), where \( A^{U \dagger} = A^U \), \( V^{U \dagger} \) is the mixing matrix for diagonalizing the mass matrix of upper-type quarks and \( \tilde{m} \) is the average squark mass in the super-KM basis. In general, the trilinear SUSY soft breaking \( A^Q \) terms are not diagonal matrices. However, due to the relation of \( A^{Q \dagger}_{ij} = (Y^Q A^Q)_{ij} \) with \( Y^Q \) \cite{20}, \( A^Q \) being Yukawa (\( A \)) parameter matrices and the small effect of renormalization group, dominant effects of \( A^Q \) are still from the diagonal elements \cite{22} if we take \( A^Q \) to be universal and diagonal at the grand unified scale. We use \( A_Q = A^Q \) to simplify our estimations. Therefore, the contribution in Fig. 1 is proportional to \( m_\mu m_a \mu^* \tan \beta (\delta_{33}^U)_{LR} \). Since \( A^U \) is hermitian, \( A^U \tilde{(\mu_\gamma^U)}_{LR} \) can be regarded as real values. Hence, in our mechanism, the CP violating source is focused on the complex \( \mu \) term. We note that by adopting a large \( \tan \beta \), the \( \mu \)-dependent effect is from the vertex of the charged Higgs coupling to squarks. For convenience, we write the relationship between weak and physical eigenstates for the mixing of \( \tilde{t}_L \) and \( \tilde{t}_R \) as

\[
\begin{pmatrix} \tilde{t}_L \\ \tilde{t}_R \end{pmatrix} = \begin{pmatrix} \cos \theta_t & \sin \theta_t \\ -\sin \theta_t & \cos \theta_t \end{pmatrix} \begin{pmatrix} \tilde{t}_1 \\ \tilde{t}_2 \end{pmatrix}.
\]

To study Fig. 1 we start with the effective interactions for quark-gluino-squark, given by \cite{27}

\[
\mathcal{L}_{\tilde{g}\tilde{q}q} = -\sqrt{2} g_s (\bar{s}_R \tilde{g}^a T^a \tilde{s}_L - \bar{b}_P \tilde{g}^a T^a \tilde{b}_R) + h.c.,
\]

where the flavor mixings for squarks have been neglected.

It is interesting to note that if we use the photon instead of the gluon and include the emission of the photon at the charged Higgs, we find that the same mechanism could also contribute to \( b \to s \gamma \). Therefore, sizable values for both \( \Delta S_{\gamma} \) and the rate CPA in \( B \to X_{\gamma} \gamma \) can definitely provide a hint for new physics. The effective operators for \( b \to s \gamma (g) \) are given by

\[
\mathcal{L} = \frac{G_F}{\sqrt{2}} V_{tb}^* V_{tb} \left[ C_{\gamma \gamma} (\mu) \mathcal{O}_{\gamma \gamma} + C_{8g} (\mu) \mathcal{O}_{8g} \right],
\]

where \( \mathcal{O}_{\gamma \gamma} = m_b \mu_e/(8\pi^2) \bar{s} \sigma_{\mu \nu} F_{\mu \nu} (1 + \gamma_5) b, \mathcal{O}_{8g} = m_b g_s/(8\pi^2) \bar{s} \sigma_{\mu \nu} T^a G_{\mu \nu} (1 + \gamma_5) b, \)

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{fig1.png}
\caption{Two-loop diagram by the \( \tilde{b}_R - \tilde{s}_L \) flavor changing effect and the chromodipole operator.}
\end{figure}
\[ C_{7\gamma} = -\cos \theta_{i} \sin \theta_{i} \frac{\tilde{V}_{tb}^{\dagger} \tilde{V}_{tb} \alpha_{s}(m_{b})}{V_{tb}^{\dagger} V_{tb}} m_{t} m_{b} \tan \beta \mu_{F}^{*} I \left( \frac{m_{q}^{2}}{m_{g}^{2}}, \frac{m_{q}^{2}}{m_{g}^{2}}, \frac{m_{H}^{2}}{m_{g}^{2}} \right), \]

\[ I \left( \frac{m_{q}^{2}}{m_{g}^{2}}, \frac{m_{q}^{2}}{m_{g}^{2}}, \frac{m_{H}^{2}}{m_{g}^{2}} \right) = m_{q}^{2} \int_{0}^{1} dx \int_{0}^{\infty} dQ^{2} \left( Q^{2} + m_{q}^{2} \right) \left( Q^{2} + m_{H}^{2} \right) \left( Q^{2} + m_{g}^{2} \right), \]

\[ C_{9q} = C_{9\gamma}/(2N C_{F} F_{K}^{2}) \text{ and } P_{\gamma}^{D} = C_{F}(Q_{U} - 1) \text{ with } C_{F} = 4/3 \text{ and } Q_{U} = 2/3 \text{ being the color factor and the charge of the upper-type squark, respectively. Clearly, we obtain the unique property that the effects of electric and magnetic dipole moments are directly related to those of chromoelectric and chromomagnetic dipole moments, respectively. Before we proceed further, we have to examine whether the two-loop effects are of interest. Explicitly, we would like to check whether the value of } C_{7\gamma} \text{ is larger or smaller than experimental constraint } 0.3 < |C_{7\gamma}| < 0.34. \]

For an illustration, we set the values of parameters, by satisfying the constraints from the NEDM \(^{29}\), as follows: tan \( \beta \sim m_{t}/m_{b} \), sin \( \theta_{b} \) cos \( \theta_{b} \sim 0.2 \), \( V_{tb}^{\dagger} V_{tb}^{\dagger} \sim O(1) \), \( m_{t}/m_{b} \sim 4 \), \( Arg(\mu) = \pi/2 \), \( m_{H} \sim 150 \text{ GeV} \), \( m_{\tilde{g}} \sim 1 \text{ TeV} \), \( m_{1} \sim 200 \text{ GeV} \) and \( m_{1} \sim 500 \text{ GeV} \), and we have \( C_{7\gamma} \sim 0.8 \). If we take tan \( \beta \sim 50 \), sin \( \theta_{b} \) cos \( \theta_{b} \sim 0.35 \), \( A_{b}/m_{\tilde{g}} \sim \mu/m_{\tilde{g}} \sim O(1) \) and the remain to be the same as the above choices, we obtain \( C_{7\gamma} \sim 0.14 \). Furthermore, by using b-quark and sbottom instead of s-quark and its squark, the similar two-loop diagram could contribute to the EDM of s-quark. It is known that the current limit of the s-quark chromo EDM is \(|ed_{s}^{C}|_{\text{expt}} < 5.4 \times 10^{-25} \text{ e cm} \). \(^{31}\) We now examine the contribution to \( d_{s}^{C} \) in our mechanism. By using Eq. \(^{5}\) and assuming \( m_{s} = m_{\tilde{g}} \) and \( A_{s} = A_{b} \), we obtain

\[ |d_{s}^{C}| \sim \sqrt{2} G_{F} V_{tb}^{\dagger} m_{s} m_{\tilde{g}} ImC_{9q}. \]

Numerically, we get \(|ed_{s}^{C}| \sim 2.5 \times 10^{-25} |C_{7\gamma}| \text{ e cm} \), which is below \(|ed_{s}^{C}|_{\text{expt}} \text{ if } |C_{7\gamma}| \leq 1 \). Clearly, in our mechanism it is inevitable to utilize the large tan \( \beta \), \( A_{b}/m_{\tilde{g}} \text{ and } m_{\tilde{g}}/m_{\tilde{g}} \) scheme and, therefore, the most strict constraint is the BR of \( B \rightarrow X_{\gamma} \).

In order to discuss the mixing-induced CP problem in \( B \rightarrow \phi K_{s} \), we write the relevant definition of the time-dependent CPA as

\[ A_{CP} = BR(B \rightarrow \phi K_{s}) - BR(B \rightarrow \phi K_{s}) \),

\[ = C_{\phi K_{s}} \cos \Delta m_{B}t + S_{\phi K_{s}} \sin \Delta m_{B}t, \]

\[ = \frac{|\lambda|^{2} - 1}{|\lambda|^{2} + 1} \cos \Delta m_{B}t - \frac{2i\lambda}{|\lambda|^{2} + 1} \sin \Delta m_{B}t, \]

where \( \lambda = e^{i\phi_{N_{ew}}} C_{7\gamma}, \) the decay amplitude. Since the dipole operators contributing to the nonleptonic decays belong to next-to-leading order in \( \alpha_{s} \), we can safely neglect the contributions to the decay amplitude of \( B \rightarrow J/\Psi K_{s} \). For displaying the other SUSY effects on the \( B \rightarrow \bar{B} \) mixing, we use \( \phi_{N_{ew}} \) instead of \( \phi_{1} \). Hence, \( \phi_{N_{ew}} \) is still determined by \( B \rightarrow J/\Psi K_{s} \), exclusively. For estimating the hadronic matrix element of \( B \rightarrow \phi K \), we use the naive factorization, given by

\[ \langle \phi K | O_{8g} | \bar{B}, p_{B} \rangle \sim -\frac{2\alpha_{s} m_{b}^{2}}{9 \pi q^{2}} f_{\phi} m_{\phi} F^{BK}(0) \epsilon^{*} \cdot p_{B}, \]

where \( F^{BK}(0) \) is the transition form factor of \( B \rightarrow K \) at \( Q^{2} = 0, q^{2} \text{ is the squared momentum of the virtual gluon, } f_{\phi} \text{ and } m_{\phi} \text{ correspond to the polarization vector, decay constant and the mass of } \phi, \) respectively. The dominant contribution of factorization assumption is confirmed by the PQCD approach \(^{32}\) in which \( q^{2} \) is related to the momentum fractions of quarks and convolutes with wave functions. We note that although \( C_{7\gamma} \) can also contribute to the decay of \( B \rightarrow \phi K_{s} \), since the coupling is electromagnetic interaction and much smaller than that of strong interaction, we neglect its contribution. Accordingly, the decay amplitude for \( B \rightarrow \phi K_{0} \) is written as

\[ A(\bar{B} \rightarrow \phi K_{0}) = \frac{G_{F}}{2} V_{tb}^{\dagger} \frac{m_{s}}{m_{t}} \frac{C_{9q}}{8 \pi^{2}} ImC_{8g}. \]

where \( a_{i} \text{, defined in Ref. } \(^{33}\), stand for the effective Wilson coefficients in the SM, included from electromagnetic penguin diagrams. The value of \( \sum_{i=3}^{5} a_{i} \text{ is estimated to be } -0.045 \). The parameter \( \lambda \) in Eq. \(^{4}\) for the CPA can be simplified as \( \lambda = e^{-i \phi_{N_{ew}}(J/\Psi K_{s})} e^{-i \phi_{N_{ew}}} \) with

\[ \tan \phi_{N_{ew}} = -\frac{2\alpha_{s} m_{b}^{2}}{9 \pi q^{2}} \sum_{i=3}^{5} a_{i} \frac{ImC_{8g}}{2 \alpha_{s} m_{b}^{2} \Re C_{8g}}. \]

To display the unique character of the two-loop diagram, we adopt the value of \( C_{7\gamma} \) such that \( C_{7\gamma} = -C_{\phi K_{s}}^{SM} \pm i|ImC_{7\gamma}^{SM}| \) and the experimental value \( C_{7\gamma}^{SM} = C_{\phi K_{s}}^{SM} \) is scanned over the whole parameter space. If \( C_{7\gamma}^{SM} = -0.30 \) and the identity \( C_{8g} = -3C_{\gamma}/8 \), the CP violating phase from the decay amplitude is tan \( \phi_{N_{ew}} = \mp(0.18 \pm 0.01 ^{+0.09}_{-0.06}) \), in which
the first error is from $|C_{\gamma}^{\text{eff}}| = 0.32 \pm 0.02$ and the second theoretical error arises from the uncertainty in $q^2 = (3/8 \pm 1/8)m_B^2$. Since $S_{\phi K_s} = 2 \phi_{\gamma}^0 (J/\Psi K_s)$, by taking $\sin 2\phi_{\gamma}^0 (J/\Psi K_s) \approx 0.74$ measured by Belle and Babar, we obtain $S_{\phi K_s} = 0.46 \pm 0.01^{+0.21}_{-0.21}(0.93 \pm 0.01^{0.05}_{-0.05})$ where the sign of $\phi_{\gamma}$ is chosen to be negative (positive). Interestingly, the former value is close to the central value of the Babar’s result $\sin 2\phi_{\gamma}^0 = 0.46$. Furthermore, we can straightforwardly calculate the difference of the CPAs to be

$$\Delta_{\phi_{\gamma}^0} = \sin 2\phi_{\gamma}^0 (J/\Psi K_s) - \sin 2\phi_{\gamma}^0 (\phi K_s)$$

$$\approx \begin{cases} 
0.28 \pm 0.01^{+0.21}_{-0.21} (-), \\
-(0.20 \pm 0.01^{+0.05}_{-0.05}) (+).
\end{cases} \tag{11}$$

We now consider the two-loop effects for the CPA in $b \rightarrow s\gamma$. According to the formalism shown in Ref. [34], the rate CPA for $b \rightarrow s\gamma$ is given by

$$A_{CP}(b \rightarrow s\gamma) \approx \frac{1}{100 |C_{\gamma}^{\text{eff}}|^2} \left\{ 1.1 \text{Im} C_2 C_{\gamma}^{\text{eff}} + 9.52 \text{Im} C_{\gamma}^{\text{eff}} C_{\gamma}^{\text{eff}} + 0.16 \text{Im} C_2 C_{\gamma}^{\text{eff}} \right\},$$

where $C_2 \approx 1.11$ and $C_{\gamma}^{\text{eff}} = C_{\gamma}^{SM} + C_{\gamma}$. With the same $C_{\gamma}$ used above, we get $A_{CP}(b \rightarrow s\gamma) \approx (10.5 \pm 0.6)\%$ for negative and positive signs in $\text{Im} C_{\gamma}$, respectively. Comparing to the recent Babar’s limit of $-0.06 < A_{CP}(b \rightarrow s\gamma) < +0.11$ [35], we find that only the result with negative sign in Eq. (11) is reliable, which could be used to resolve the sign ambiguity in $\text{Im} C_{\gamma}$. Finally, we remark that although our upper value on the CP asymmetry of $b \rightarrow s\gamma$ is a little bit over the Babar upper bound, the problem can be removed by relaxing the required condition $C_{\gamma} = -C_{\gamma}^{SM} \pm i|\text{Im} C_{\gamma}^{\text{eff}}|$ introduced for our simplified analysis.

In summary, we have studied the novel two-loop SUSY effects on the CPAs of $B \rightarrow \phi K_s$ and $b \rightarrow s\gamma$. We have found that with large values of $\tan \beta$ and $A_{\ell} (m_{\tilde{g}})$, the difference of $\sin 2\phi_{\gamma}^0$ between $J/\Psi K_s$ and $\phi K_s$ can have a deviation of $20 - 50\%$. The main theoretical error is due to the uncertainty in $q^2$. We have also shown that the two-loop effect can give the CPA in $b \rightarrow s\gamma$ around $+10\%$. It is clear that, since the two-loop contributions to the CPAs in both decay modes can be the dominant ones in the SUSY models, experimental measurements at $B$ factories on these CPAs can determine the sizes of these novel contributions.

**Note added**: Our two-loop SUSY mechanism has been applied to the decay of $B_s \rightarrow \mu^+ \mu^-$ [36].

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