Effects of design dimensions on the driving force variation of a hybrid TRR–XY parallel kinematic machine tool

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Abstract: Using parallel-link mechanism as the basic structure of the parallel kinematic machine (PKM) tool is a new design concept and has become one of the most important research fields, attracting many previous researchers. Research on the actuator driving force variation of a PKM tool is essential for the applications of a PKM tool to real machining processes. For machining a component, the smaller actuator driving force required means that the PKM tool has a higher machining performance. To improve the machining performance, understanding of the required actuator driving is very important. A generalized force analysis of the developed system was obtained using the Denavit–Hartenberg (D–H) notation method. The force reactions on the joints and actuator driving forces are obtained on the basis of various cutter locations and a given external cutting force. The prediction of the cutter location with the singularity condition can also be obtained on the basis of the infinite force of the joints. The geometry relationship between the tool platform and the links is adopted to verify the singularity conditions of the system. The driving force variation of the hybrid TRR–XY five-degree-of-freedom machine tool is further investigated.

Keywords: driving force, machine tool, parallel kinematic, force, singularity

NOTATION

\[ A_p^{p-1} \] (a = z, u, v) D–H coordinate transformation matrix between coordinates \( p - 1 \) and \( p \), for the \( a = z, u \) and \( v \) chains

\( B_1, B_2, B_3 \) locations of the three ball joints

\( C_1, C_2, C_3 \) centre locations of the three pin joints

\( F_{ext} \) cutting force (N)

\( F_{ix} \) (\( i = z, u, v \)) forces on \( B_1, B_2 \) and \( B_3 \) respectively along the corresponding link axis (N)

\( F_{jz} \) (\( j = z, u, v \)) forces on \( B_1, B_2 \) and \( B_3 \) respectively parallel to the corresponding pin joint axis (N)

\( i_p, j_p, k_p \) unit vectors based on the \( (X, Y, Z)_p \) (\( p = 0–7 \)) coordinate system

\( j_L \) unit vector of the cutter feed direction at the cutting point

\( k_L \) normal vector of the part surface at the cutting point

\( L \) link length (mm)

\( P_{x}, P_{y}, P_{z} \) translation distances of the origin point of \( (X, Y, Z)_L \) related to the origin point of \( (X, Y, Z)_0 \) (mm)

\( P_{0L} \) distance of the origin point of \( (X, Y, Z)_L \) relative to the origin point of the \( (X, Y, Z)_0 \) coordinate system (mm)

\( r \) radius of the tool platform (mm)

\( r_u, r_v, r_z \) \( O_7B_2 \) expressed in the coordinate system \( (X, Y, Z)_7 \)

\( r_u, r_v, r_z \) \( O_7B_3 \) expressed in the coordinate system \( (X, Y, Z)_7 \)

\( r_u, r_v, r_z \) \( O_7B_1 \) expressed in the coordinate system \( (X, Y, Z)_7 \)

\( R \) radius of the base frame (mm)

\( S_z, S_{u}, S_{v} \) distances between the origin points of \( (X, Y, Z)_0 \) and \( (X, Y, Z)_z \) on the \( z \) chain, \( u \) chain and \( v \) chain respectively

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1 INTRODUCTION

Using the parallel kinematic mechanism as the basic structure of the parallel kinematic machine (PKM) tool is a new design concept and has attracted many previous researchers in recent years [1–4]. The announcement of the Variax of Giddings and Lewis and the Hexapod of Ingersoll at the International Machine Tool Show (Chicago, 1994) show that the configuration of the machine tool may abandon conventional solutions and may lead to interesting alternatives. However, the performance of this new type of PKM tool in real machining applications has not yet been proven to be better than the conventional five-axis machines. From the design viewpoint, a PKM tool is more difficult to analyse. The reasons are as follows:

1. The kinematic and dynamic analysis of the closed chain mechanism is more complicated than that of the open orthogonal mechanism.
2. The volume of working space of a PKM tool is irregular and is more difficult to describe. The singularity condition inside the workspace is also very difficult to predict.
3. The motion constraints of different joints make the analysis and calculation of the workspace more complicated.

The machining performance of the system is another important subject that plays an important role in the design or development of the system and must be carefully considered before the system can be applied to the real machining applications. A systematic analysis of the machining performance of a PKM tool is urgently needed by industry. Many previous research results, however, are focused on the kinematics of a parallel mechanism only. Machining performance is the ratio of the input actuator driving force to the payload or external cutting force. Therefore, the analysis of the driving force variation is very important for understanding the machining performance of the developed machine tool system. Using several assumptions, the force analysis for key components and actuators are focused on in this research. Based on the cutting force and cutting moment that generated during cutting processes, the force equations developed in this research can be used to understand the force variations of the joints and links at the various cutter locations. The actuator driving force variations and the singularity conditions can be further obtained from the results obtained. Figure 1 shows the hybrid structure of the PKM tool that is developed in this research. The hybrid structure is composed of a fully parallel-link mechanism with three degrees of freedom (DOFs) and a two-DOF XY table to take advantage of both the parallel structure and open orthogonal structure. The XY table is used to increase the workspace of the system. The parallel-link mechanism has a translation and two rotational DOFs (TRR). Compared with the six-DOF, fully PKM tool, this hybrid structure is much simpler and easier to control.

2 THEORY

The driving force analysis of each driving axis is the first step for understanding the machining performance of the system. The actuator driving force of each driving axis is obtained by using the force balance theory and is based on the geometric relationship between the driving axes, tool platform and links. To simplify the calculation, the following assumptions are used:

1. The thermal effects and acceleration effects (dynamic behaviour) are neglected.
2. The inertia forces and moments are neglected.
3. The frictional forces are neglected.
4. All the components are considered to be rigid.

The main character of the developed system is the three driving axes which are evenly located on the circle with radius R (see Fig. 1). Three links, one end with a one-DOF pin joint and the other joint with a three-DOF ball joint, are used to connect the driving axis and the tool platform. The link length is L and the tool length is t. Here, t is the distance between the tool tip O7, and the centroid of triangle that formed by the three ball joints B1, B2 and B3 (see Fig. 1). As the pin joints are used, the angle between each link is 120° from the top view. The angle between link and driving axis is defined as \( \theta_a (a = z, u, v) \). The Denavit–Hartenberg (D–H)
notation method is adopted in this research. The translation and rotation of coordinates are essential in the solving procedures using the D–H notation method \[3, 5\]. The definitions of the coordinate systems of the PKM tool are shown in Fig. 2. The free-body diagram of the tool platform is shown in Fig. 3. The locations of the three ball joints are defined as \(B_1\), \(B_2\) and \(B_3\). The centre locations of the three pin joints are defined as \(C_1\), \(C_2\) and \(C_3\). In Fig. 3, the forces and moments that are generated from the cutting processes are defined as \(F_{\text{ext}}\). \(i\hat{F}_x\), \(j\hat{F}_y\) and \(k\hat{F}_z\) in the definition of \(F_{\text{ext}}\) are unit vectors based on the \(X, Y, Z_0\) coordinate system. The forces generated on the ball joints from the tool platform are defined as \(F_{xx}\), \(F_{xz}\), \(F_{ux}\), \(F_{uz}\), \(F_{vx}\) and \(F_{vz}\). \(i\hat{F}_x\), \(u\hat{F}_u\) and \(v\hat{F}_v\) are defined as the forces on \(B_1\), \(B_2\) and \(B_3\) along the corresponding link axis. \(F_{zx}\) \((j = z, u, v)\) are defined as the forces on \(B_1\), \(B_2\) and \(B_3\) parallel to the corresponding direction of the pin joint axis.

The coordinate systems defined in Fig. 2 are \((X, Y, Z)_p\) (\(f = z, u, v; p = 1–6\)). The coordinate transformation in the D–H notation method between two coordinate systems \(p – 1\) and \(p\) is defined as \(A_p^{p-1}\) \((a = z, u, v)\). \(A_p^{p-1}\) represents the transformation matrix between \((X, Y, Z)_{zp}\) and \((X, Y, Z)_{z(p-1)}\) on the \(z\) chain. The \(z\) chain is defined as the kinematic chain from \((X, Y, Z)_0\) through \(C_1, B_1\) to \((X, Y, Z)_7\) (see Fig. 1). A similar definition is applied to the \(u\) chain and the \(v\) chain. The parameters of the D–H transformation matrix between \((X, Y, Z)_0\) and \((X, Y, Z)_3\) are listed in Table 1. In Table 1, \(S_z\), \(S_u\) and \(S_v\) represent the distances between the origin points of \((X, Y, Z)_1\) and \((X, Y, Z)_2\) on the \(z\) chain, \(u\) chain and \(v\) chain respectively. Using the homogeneous transformation matrix method, the...
The transformation matrix $A^{p-1}_p$ can be given as

$$A^{p-1}_p = \begin{bmatrix}
\cos(\theta_p) - \sin(\theta_p) \cos(\alpha_p) & \sin(\theta_p) \sin(\alpha_p) & a_p \cos(\theta_p) \\
\sin(\theta_p) \cos(\alpha_p) - \cos(\theta_p) \sin(\alpha_p) & \cos(\theta_p) \sin(\alpha_p) & a_p \sin(\theta_p) \\
0 & \sin(\alpha_p) & \cos(\alpha_p) \\
0 & 0 & d_p
\end{bmatrix}$$

(1)

The unit vectors along $F_x$ and $F_z$ are separately defined as $Z_x$ and $Z_z$. In Figs 2 and 3, $Z_x$ and $Z_z$ are defined as the directions of $-i_3$ and $k_3$ of the $(X,Y,Z)_0$ coordinate system. Using the D–H notation method, $Z_x$ and $Z_z$ can be obtained as

$$Z_x = A_{z, 1}^{0} A_{z, 2}^{1} A_{z, 3}^{2} (-i_3),$$

$$Z_z = A_{z, 1}^{0} A_{z, 2}^{1} A_{z, 3}^{2} k_3,$$

(2)

Table 1 Parameters of the D–H transformation matrix between $(X,Y,Z)_0$ and $(X,Y,Z)_3$

<table>
<thead>
<tr>
<th></th>
<th>$z$ chain</th>
<th></th>
<th></th>
<th></th>
<th>$u$ chain</th>
<th></th>
<th></th>
<th></th>
<th>$v$ chain</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$d_p$ (mm)</td>
<td>$\theta_p$ (deg)</td>
<td>$a_p$ (mm)</td>
<td>$\alpha_p$ (deg)</td>
<td>$d_p$ (mm)</td>
<td>$\theta_p$ (deg)</td>
<td>$a_p$ (mm)</td>
<td>$\alpha_p$ (deg)</td>
<td>$d_p$ (mm)</td>
<td>$\theta_p$ (deg)</td>
<td>$a_p$ (mm)</td>
<td>$\alpha_p$ (deg)</td>
</tr>
<tr>
<td>Link 1</td>
<td>0</td>
<td>-150</td>
<td>$R$</td>
<td>-90</td>
<td>0</td>
<td>90</td>
<td>$R$</td>
<td>-90</td>
<td>0</td>
<td>-30</td>
<td>$R$</td>
<td>-90</td>
</tr>
<tr>
<td>Link 2</td>
<td>0</td>
<td>90</td>
<td>$S_z$</td>
<td>0</td>
<td>0</td>
<td>90</td>
<td>$S_z$</td>
<td>0</td>
<td>0</td>
<td>90</td>
<td>$S_z$</td>
<td>0</td>
</tr>
<tr>
<td>Link 3</td>
<td>0</td>
<td>$\theta_e$</td>
<td>$L$</td>
<td>0</td>
<td>0</td>
<td>$\theta_e$</td>
<td>$L$</td>
<td>0</td>
<td>0</td>
<td>$\theta_e$</td>
<td>$L$</td>
<td>0</td>
</tr>
</tbody>
</table>
In the above equation, \((i_z, j_z, k_z)\) is the unit vector of \((X, Y, Z)_z\). Therefore, the force vectors \(F_{xz}\) and \(F_{zz}\) can be represented as

\[
F_{xz} = F_{xz}Z_x = F_{xz} \left[ \frac{-\sqrt{3}}{2} \sin(\theta_z) \quad -\frac{1}{2} \sin(\theta_z) \cos(\theta_z) \right]^T
\]

\[
F_{zz} = F_{zz}Z_z = F_{zz} \left[ \frac{1}{2} \frac{-\sqrt{3}}{2} \quad 0 \quad 0 \right]^T
\]

Applying similar derivations, the forces \(F_{ux}\) and \(F_{uz}\) can be obtained as

\[
F_{ux} = F_{ux} \left[ 0 \quad \sin(\theta_u) \cos(\theta_u) \quad 0 \right]^T
\]

\[
F_{uz} = F_{uz} \left[ -1 \quad 0 \quad 0 \right]^T
\]

In the above equation, \(U_x\) and \(U_z\) represent the unit vectors of the forces along \(F_{ux}\) and \(F_{uz}\) respectively on the \(u\) chain. Similarly, the forces \(F_{vx}\) and \(F_{vz}\) of the ball joint at \(B_2\) on the \(v\) chain can be obtained as

\[
F_{vx} = F_{vx} \left[ \frac{\sqrt{3}}{2} \sin(\theta_v) \quad -\frac{1}{2} \sin(\theta_v) \cos(\theta_v) \right]^T
\]

\[
F_{vz} = F_{vz} \left[ \frac{1}{2} \frac{\sqrt{3}}{2} \quad 0 \quad 0 \right]^T
\]

In the above equation, \(V_x\) and \(V_z\) represent the unit vectors of the forces along \(F_{vx}\) and \(F_{vz}\) respectively on the \(V\) chain. Using the free-body diagram in Fig. 3, the force equilibrium theory of the tool platform can be obtained as follows:

\[
\sum F_x = F_x + \left[ \frac{-\sqrt{3}}{2} F_{xz} \sin(\theta_z) \right] + \frac{F_{xz}}{2} + 0
\]

\[
\sum (-F_{ux}) + \frac{\sqrt{3}}{2} F_{vx} \sin(\theta_v) + \frac{F_{vx}}{2} = 0
\]

\[
\sum F_y = F_y + \left[ \frac{-1}{2} F_{yx} \sin(\theta_y) \right] + \left( \frac{-\sqrt{3}}{2} F_{zz} \right)
\]

\[
+ F_{uy} \sin(\theta_u) + \left[ \frac{F_{vx}}{2} \sin(\theta_v) \right] \right] + \frac{\sqrt{3}}{2} F_{xz} = 0
\]

\[
\sum F_z = F_z + F_{xz} \cos(\theta_z) + 0 + F_{ux} \cos(\theta_u)
\]

\[
+ F_{vx} \cos(\theta_v) = 0
\]

The part surface that is generated by commercial computer aided design/computer aided manufacture (CAD/CAM) software is usually represented by a cutting location (CL) data file with a format \((x, y, z, i, j, k)\) based on a local surface coordinate system \((X, Y, Z)_L\) [6]. \((x, y, z)\) is the coordinate system of the cutting point and \((i, j, k)\) is the normal unit vector of the cutting point. Then

\[
A_L^0 = \begin{bmatrix}
  i_L & j_L & k_L & P_{0L} \\
  0 & 0 & 1 & 0
\end{bmatrix}_{4 \times 4}
\]

In the above equation, \(j_L\) is the unit vector of the cutter feed direction at the cutting point. \(k_L\) is the normal direction of the part surface at the cutting point. \(k_L\) is defined as \(k_L = \hat{i}_0 + \hat{j}_0 + kk_0\) and \(i, j, k\) are generated by the CAD/CAM software. \(P_{0L} = P_{x0}i_0 + P_{y0}j_0 + P_{z0}k_0\) is the distance of the origin point of \((X, Y, Z)_L\) relative to the origin point of the \((X, Y, Z)_0\) coordinate system.

In real machining applications, the cutter geometry is very important from the machining efficiency viewpoint. In this research, a parametric tool concept is adopted to develop the generalized post-processor theory of the machine tool with non-normal cutting. There are seven geometry parameters included in the definition of the parametric tool. The physical meaning of the geometry parameters are shown in Fig. 4a [7]. In non-normal cutting, the cutting point is moved along the \(Y_L\) direction of the local \((X, Y, Z)_L\) coordinate system and \(Z_l\) is the normal direction at the cutting point (Fig. 4b). The transformation between the coordinates \((X, Y, Z)_{\gamma}\) and \((X, Y, Z)_L\) is obtained by rotating through a tilt angle \(\lambda\) about the \(X_L\) axis, followed by rotating a yaw angle \(\omega\) about the \(Z_L\) axis and followed by a translation (Fig. 4c). Figure 4d shows a translation relationship between the coordinates \((X, Y, Z)_{\gamma}\) and \((X, Y, Z)_L\). Here, the contact angle at cutting point is also defined as \(\psi\) (see Fig. 4d). Therefore, the transformation matrix between the coordinates \((X, Y, Z)_{\gamma}\) and \((X, Y, Z)_L\) is obtained as \(A_L^\gamma\):

\[
A_L^\gamma = \text{Rot}(X_L, \lambda) \text{ Rot}(Z_L, \omega) \text{ Trans}(0, -H, -V)
\]

In the above equation,

\[
H = E + r_c \cos \psi
\]

\[
V = F - r_c \sin \psi
\]

The definition of cutter location and orientation related to the global coordinate system \((X, Y, Z)_0\) are essential in this research. \(T_0^\gamma\) is the homogeneous coordinate transformation matrix from \((X, Y, Z)_{\gamma}\) to \((X, Y, Z)_0\). Using the coordinate definition in Fig. 2, \(T_0^\gamma\) can be obtained as follows:

\[
T_0^\gamma = \text{Trans}(P_x, P_y, P_z) \text{ Rot}(X_0, \alpha) \text{ Rot}(Y_0, \beta)
\]

\[
\times \text{Rot}(Z_0, \gamma)
\]

In the above equation, \(\alpha, \beta\) and \(\gamma\) are the rotational angles of the tool platform related to the axis \(X_0, Y_0, Z_0\).
and $Z_0$ respectively of the $(X, Y, Z)_0$ coordinate system. $P_x$, $P_y$, and $P_z$ are the translation distances of the origin point of $(X, Y, Z)_7$ related to the origin point of $(X, Y, Z)_0$. By applying the relationship $T_{7}^0 = A_{7}^0 A_{7}^{-1}$, $\alpha$, $\beta$, $\gamma$, $P_x$, $P_y$, and $P_z$ can be obtained. As the moment calculation is based on the $(X, Y, Z)_7$ coordinate system, the equivalent moment $M_O$ is obtained by shifting the external force $F_{\text{ext}}$ to point $O_7$. From the geometry relationship in Figs 2 and 3, the force arms $O_7B_1$, $O_7B_2$ and $O_7B_3$ can be written as

\begin{align}
\vec{r}_x = O_7B_1 &= \left[ \begin{array}{c} -\frac{\sqrt{3}r}{2} \\
- \frac{r}{2} \\
t \\
1 \end{array} \right]^T \\
\vec{r}_u = O_7B_2 &= \left[ \begin{array}{c} 0 \\
1 \\
t \end{array} \right]^T \\
\vec{r}_v = O_7B_3 &= \left[ \begin{array}{c} \frac{\sqrt{3}r}{2} \\
- \frac{r}{2} \\
t \\
1 \end{array} \right]^T
\end{align}

Fig. 4 (continued over)
In the above equation, $\mathbf{r}_z$ is the $z$ vector based on the coordinate system $(X, Y, Z)$. Therefore, $\mathbf{0}_r$, $\mathbf{0}_u$ and $\mathbf{0}_v$ can be obtained as

$$
\mathbf{0}_r = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
\mathbf{T}^0_0 \mathbf{r}_z - [P_x \ P_y \ P_z \ 1]^T
\end{bmatrix}
= [E_1 \ E_2 \ E_3]^T
$$

$$
\mathbf{0}_u = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
\mathbf{T}^0_0 \mathbf{r}_u - [P_x \ P_y \ P_z \ 1]^T
\end{bmatrix}
= [E_4 \ E_5 \ E_6]^T
$$

$$
\mathbf{0}_v = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
\mathbf{T}^0_0 \mathbf{r}_v - [P_x \ P_y \ P_z \ 1]^T
\end{bmatrix}
= [E_7 \ E_8 \ E_9]^T
$$

Variables $E_1$ to $E_9$ are used in equations (20) to (22) to reduce the complexity of the $\mathbf{0}_r$, $\mathbf{0}_u$ and $\mathbf{0}_v$ calculation. The moment generated by the cutting force can be obtained as

$$
\mathbf{M}_{\mathbf{O}_1} = \mathbf{0}_r \mathbf{O}_1 \times \mathbf{F}_{\text{ext}} = M_x \mathbf{i}_0 + M_y \mathbf{j}_0 + M_z \mathbf{k}_0
$$

$\mathbf{0}_r \mathbf{O}_1$ is defined as follows:

$$
\mathbf{0}_r \mathbf{O}_1 = \mathbf{O}_1 \mathbf{O}_L = [P_{xL} \ P_{yL} \ P_{zL}]^T - [P_x \ P_y \ P_z]^T
$$

\(F\) has already been defined in equations (3) to (8) and \(r\) has already been defined in equations (20) to (22). The following equations can be obtained by using the moment balance theory:

1. From \((r \times F) \cdot \mathbf{i}_0 + M_x = 0\),

\[
F_{zx} \left[ E_2 \cos(\theta_z) + \frac{E_3}{2} \sin(\theta_z) \right] + F_{zz} \sqrt{3} E_3 + F_{ux} E_5 \cos(\theta_u) - E_6 \sin(\theta_u)
F_{zx} \left[ E_8 \cos(\theta_v) + \frac{E_9}{2} \sin(\theta_v) \right] - F_{zz} \sqrt{3} E_9
= -M_x
\]

2. From \((r \times F) \cdot \mathbf{j}_0 + M_y = 0\),

\[
F_{zx} \left[ -E_1 \cos(\theta_z) - \frac{\sqrt{3} E_3}{2} \sin(\theta_z) \right] + F_{zz} \frac{E_3}{2} - F_{ux} E_4 \cos(\theta_u) - F_{uz} E_6
F_{zx} \left[ -E_7 \cos(\theta_v) + \frac{\sqrt{3} E_9}{2} \sin(\theta_v) \right] + F_{zz} \frac{E_9}{2}
= -M_y
\]
3. From \((r \times F) \cdot k_0 + M_z = 0\),
\[
F_{xz} \left[ -\frac{E_1}{2} \sin(\theta z) + \frac{\sqrt{3}E_2}{2} \sin(\theta z) \right] + F_{zz} \left( -\frac{\sqrt{3}E_1}{2} \right) + F_{ux}E_4 \sin(\theta_u) + F_{uz}E_5 \\
+ F_{vx} \left[ -\frac{E_7}{2} \sin(\theta_v) - \frac{\sqrt{3}E_8}{2} \sin(\theta_v) \right] + F_{vz} \left( -\frac{\sqrt{3}E_7}{2} \frac{E_8}{2} \right) = -M_z
\]
(27)

The forces \(F_{xz}, F_{zz}, F_{ux}, F_{vz}, F_{ux}, F_{vz}\), and \(F_{ux}\), can be solved by simultaneously solving the tool platform force equilibrium [equations (9) to (11)] and the moment equilibrium [equations (25) to (27)]. The resultant forces on the ball joints at \(B_1, B_2\), and \(B_3\) and pin joints at \(C_1, C_2\), and \(C_3\) generated in the cutting processes can then be solved.

3 DESIGN OF PARAMETERS

The effects of the design dimensions on the forces generated on the ball joints and pin joints during the cutting processes with various cutter locations and orientations will be investigated in this section. Here, the external force \(F_{ext}\) is applied on the tip of the cutter \((O_7)\). The equivalent moment \(M_{O_7}\) generated by the external force is considered to be constant and along the tool axis direction. To increase the generality of the analysis results, dimensionless parameters are adopted. First, a group of design parameters are selected as the comparison reference. They are \(r/R = 0.8, t/R = 1.08\) and \(L/R = 2.4\). The following interesting and important factors are separately discussed.

3.1 Effects of cutter orientations

The definitions of the cutter locations and orientations are shown in Fig. 5. In Fig. 5, \(O_7\) of the spherical coordinate system coincides with \(O_7\). The orientation of the coordinate system \((X, Y, Z)\) is the same as that of \((X, Y, Z)\). The cutter vector \(t\) is defined as
\[
t = t \sin(\phi) \cos(\varphi)i_0 + t \sin(\phi) \sin(\varphi)j_0 \\
+ t \cos(\phi)k_0
\]
(28)

In the above equation, \(\phi\) is the angle between cutter vector and \(Z\) axis, \(\varphi\) is the angle between the \(X\) axis and the cutter axis vector projected on the \(XY\) plane. It is worth mentioning that the definitions of \(\phi\) and \(\varphi\) are based on the right-hand rule. In this research, the force variation is investigated within the whole domain \(\phi = -80^\circ\) to \(80^\circ\) and \(\varphi = 0\) to \(180^\circ\). The singularity can also be analysed from the obtained force analysis results.

3.2 Effects of cutting forces and moments

The external cutting forces and moments are varied to investigate the variation in the force condition of the ball joints and pin joints. The singularity variation will also be examined in this section. The structure design parameters of the machine tool are the same as in Section 3.1. The cutting forces and moments are planned to be as follows:

(a) \(F_{ext} = 100i_0 + 100j_0 + 100k_0 N\) and \(M_{O_7} = 10000k_7 Nmm\).

(b) \(F_{ext} = 0i_0 + 100j_0 + 100k_0 N\) and \(M_{O_7} = 0 Nmm\).

(c) \(F_{ext} = 0i_0 + 0j_0 + 100k_0 N\) and \(M_{O_7} = 0 Nmm\).

(d) \(F_{ext} = 0i_0 + 0j_0 + 0k_0 N\) and \(M_{O_7} = 0 Nmm\).

3.3 Effects of various \(r/R\)

\(R\) represents the size of the machine tool and \(r\) represents the size of the tool platform. In this research, \(R\) is fixed and \(r\) is varied in the ratios \(r/R = 0.8, 0.4\) and \(0.2\) to investigate the effects of various \(r/R\) on the force condition of ball joints and pin joints. Here, the other parameters are kept the same as the parameters in Section 3.2 \((t/R = 1.08\) and \(L/R = 2.4\)).

3.4 Effects of various \(L/R\)

With fixed \(t/R = 1.08\) and \(r/R = 0.8\), the \(L/R\) ratio is varied as follows to investigate the effects: \(L/R = 2, 3\) and 4.
Fig. 6  Force and moment variations in the ball joints $B_1$, $B_2$ and $B_3$ and pin joints $C_1$, $C_2$ and $C_3$ with $r/R = 0.8$, $t/R = 1.08$ and $L/R = 2.4$. 
3.5 Effects of various $r/R$ on the actuator driving force

The analysis of the actuator driving force required to complete the machining processes of the machine tool is very important for understanding the machining performance of the machine tool. In the open-chain, orthogonal and serial-type machine tool, the machining performance is defined as

\[
\xi_{x,y,z} = \frac{X, Y, Z \text{ axes of } (X, Y, Z)_0}{\text{cutting force on cutter tip relative to } X, Y, Z \text{ axes of } (X, Y, Z)_0}
\]  (29)

The above definition is very easy to understand. In the PKM tool, it is, however, more difficult to understand. Based on the PKM tool developed in this research, the three driving axes are all arranged on the $Z$ (vertical) direction. Therefore, the machining performance of the machine tool is defined as

\[
\xi = \frac{\text{total cutting forces on the cutter}}{\sum (\text{actuator driving forces for the } Z_i \text{ axis}, i = 1-3)}
\]  (30)

The effects of various $r/R$ on the machining performance $\xi$ are then investigated. The workspace domain of the cutter orientation variation analysed in this research is set as $\phi = -60^\circ$ to $60^\circ$ and $\varphi = 45^\circ$. The forces generated on the cutter are set as $F_{\text{ext}} = 100k_0 \text{ N}$ and $M_{\text{O}_0} = 10000k_0 \text{ N mm}$. $r/R$ is varied as follows: $r/R = 0.8$, $0.4$ and $0.2$. Here, $t/R = 1.08$ and $L/R = 4$ are fixed.

3.6 Effects of various $L/R$ on the actuator driving forces

The parameters $r/R$ and $t/R$ are fixed. The cutter orientation and cutting force are the same as in Section 3.5. The $L/R$ is varied as follows: $L/R = 2$, $3$ and $4$.

4 SOLUTION APPROACH

The steps of the solution procedures are briefly described as follows:

1. Input the tool path data, and the variation in workspace domain ($\alpha$ and $\beta$) can be obtained. The tool path data can be obtained from the theoretical derivation or CL data file from CAD/CAM software. The angles $\alpha$ and $\beta$ can be obtained from the unit vector $(i, j, k)$ of the cutter orientation vector.
2. Input the machine tool dimensions $r$, $t$, $R$ and $L$ and the cutting force $F_{\text{ext}}$ and moment $M_{\text{O}_0}$.
3. Calculate the homogeneous and D–H transformation matrix based on the coordinate system shown in Fig. 2. The transformation relationship of the forces related to the different coordinate system can be obtained.
4. $F_{xz}$, $F_{zz}$, $F_{ux}$, $F_{uz}$, $F_{vx}$, $F_{vz}$ and $r \times F$ ($i = z, u$ and $v$) are decomposed into three unit vector directions $(i_0, j_0, k_0)$. The six force equilibrium equations are calculated and used to solve the six force variables.
5. The force results on $B_1$, $B_2$, $B_3$, $C_1$, $C_2$ and $C_3$ can be obtained by multiplication of the D–H matrix $T_0^i$ and the forces $F_{xz}$, $F_{zz}$, $F_{ux}$, $F_{uz}$, $F_{vx}$ and $F_{vz}$ (obtained in step 4). Output the force analysis results of ball joints and pin joints.
6. For the singular position, one of the resultant forces of the joints may be increased to be very large (infinite). The singular position obtained from the force analysis is further examined by the geometry relationship of the machine tool.

5 RESULTS AND DISCUSSION

The force analysis results can be obtained after the theoretical derivation and the numerical calculation. Several interesting results were found and are discussed in the following:

1. There is a ‘safe workspace’ found in Fig. 6 based on the force and moment analysis results of ball joints $B_1$, $B_2$ and $B_3$ and pin joints $C_1$, $C_2$ and $C_3$. The safe workspace is of conic shape and has $\phi = \pm 50^\circ$...
Fig. 8  Force variation in the ball joint $B_1$ and moment variation in the pin joint $C_1$ with different cutting forces ($r/R = 0.8$, $t/R = 1.08$ and $L/R = 2.4$)
Fig. 9  Force variation in the ball joint $B_1$ and moment variation in the pin joint $C_1$ with different $r/R$ ratios ($r/R = 1.08$ and $L/R = 2.4$)
and $\varphi = 0^\circ - 180^\circ$. Here, the safe workspace is defined as when the force on the ball joints is smaller than 400 N and the moment on the pin joint is smaller than 80,000 N mm. The forces increased very rapidly with increasing cutter orientation. The singular position is also located in this region. In Fig. 6, it is found that the singular position is an ‘arc-shaped’ area and located in the region of large cutter orientations. One of the geometry relationships of the singular position is shown in Fig. 7. For this case $\phi = 67.115^\circ$, $\varphi = 30^\circ$ and $\theta_z$, $\theta_u$ and $\theta_v$ are calculated to be 22.88$^\circ$, 4.78$^\circ$ and 4.78$^\circ$ respectively. It is found that the link $Z$ is coplanar with the plane of the tool platform. In this situation, link $Z$ cannot support the cutting force along the cutter axis. A very large force is required to balance the cutting force along $Z$.

2. Figure 8 shows the force analysis results of the ball joints and pin joints with different cutting forces. It is found that the effect of $M_{\Omega_1} = 10,000 k_N$ mm is much smaller than the effect of $F_{\text{ext}} = 100 i_0$ N within the domain $\phi = \pm 50^\circ$, $\varphi = 0^\circ - 180^\circ$. With $\phi > \pm 50^\circ$ and $\varphi = 0^\circ - 180^\circ$, the forces of the joints increase very rapidly with the position close to the singular position. The forces decrease after passing the singular position. In Figs 8e and f, only the moment is applied to the cutter; the distribution of the singular position is very regular with an arc shape. Another interesting result is that the singular position of the machine tool is not varied when the external cutting force is varied.

3. Compare the results in Figs 9a, b and c; it is found that the forces on the joints vary approximately in the ratio 1:2:4 when $r/R$ is varied from $r/R = 0.8$ to 0.4 to 0.2. This implies that, the smaller the $r/R$ ratio, the larger are the joint forces that are encountered. From the results in Figs 9d, e and f, it is found that the moment variation on the joints is not very significant when $r/R$ is significantly varied. For the singular position, the tool path with $\phi = -80^\circ$ to 80$^\circ$ and $\varphi = 30^\circ$ is used to explain the effects of the ratio $r/R$. In Fig. 10, the singular position is found to be very slightly decreased with the $r/R$ ratio decreased. This implies that the workspace is also slightly decreased. The singular position is obtained as $\phi = 67.115^\circ$, $\phi = 66.425^\circ$ and $\phi = 65.955^\circ$ for $r/R = 0.8$, $r/R = 0.4$ and $r/R = 0.2$ respectively.

4. The effects of $L/R$ on the forces on the joints are shown in Fig. 11. The forces on the joints are found in Fig. 10 to be slightly decreased when the $L/R$ ratio is varied from $L/R = 2$ to 3 to 4. However, the moment of the joints are slightly increased with the same $L/R$ ratio variation. The tool path in the preceding result in Section 4 is used again to investigate the singular position. In Fig. 12, the singular position is found to increase with increasing $L/R$ ratio ($L/R = 2$, 3 and 4). The singular position was found to be $\phi = 64.055^\circ$, $\phi = 70.529^\circ$ and $\phi = 74.382^\circ$ for $L/R = 2$, $L/R = 3$ and $L/R = 4$ respectively. Therefore, increasing the link length of the machine tool may increase the cutter workspace (cutter orientation).
Fig. 11  Force variation in the ball joint B\textsubscript{1} and moment variation in the pin joint C\textsubscript{1} with different $L/R$ ratios ($r/R = 0.8$ and $t/R = 1.08$)
5. A special tool orientation variation arrangement with $\phi = -60^\circ$ to $60^\circ$ and $\varphi = 45^\circ$ is used to show the effects of various $r/R$ ratios on the variation in the actuator driving forces of the machine tool. Figures 13, 14, 15 and 16 show the effects of $r/R$ on the driving axis variation with external cutting forces $F_{\text{ext}} = 100k_\theta N$, $F_{\text{ext}} = 100j_\theta N$, $F_{\text{ext}} = 100k_\theta N$ and $M_{O_7} = 10000k_7 N \text{mm}$ respectively. By comparing Figs 13 to 16, the following results can be obtained:

(a) The required actuator driving forces for an $XY$ plane force applied to the cutter is much larger than that for a force being applied to the cutter along the $Z_0$ direction ($F_{\text{ext}} = 100k_\theta N$) or for a moment applied along the tool axis ($M_{O_7} = 10000k_7 N \text{mm}$).

(b) With the cutting force $F_{\text{ext}} = 100k_\theta N$, no exact relationship is found between the $r/R$ variation
and the actuator driving force variation. With the other cutting force conditions $F_{\text{ext}} = 100i_0 \, \text{N}$, $F_{\text{ext}} = 100j_0 \, \text{N}$ and $M_O = 10000k_7 \, \text{Nmm}$), an inversely proportional relationship is found between the $r/R$ variation and the actuator driving force variation. This means that, the smaller the $r/R$ ratio, the larger are the required actuator driving forces. The actuator driving force variation with $F_{\text{ext}} = 100k_0 \, \text{N}$ are summarized as follows:

(i) For link $z$, the required actuator driving force for the case $r/R = 0.8$ is larger than for $r/R = 0.2$ within the range $\phi = -60^\circ$ to $0^\circ$. For the range $\phi = 0^\circ$ to $60^\circ$, the result is the opposite.

(ii) For link $u$, the required actuator driving force for the case $r/R = 0.8$ is smaller than for $r/R = 0.2$ within the range $\phi = -60^\circ$ to $0^\circ$. For the range $\phi = 0^\circ$ to $60^\circ$, the result is the opposite.

(iii) For link $v$, the required actuator driving force for the case $r/R = 0.8$ is larger than for $r/R = 0.2$ within the full range $\phi = -60^\circ$ to $-60^\circ$. 

Fig. 14 Variation in actuator driving force with different $r/R$ ratios ($F = 100j \, \text{N}$, $r/R = 1.08$ and $L/R = 2.4$)

Fig. 15 Variation in actuator driving force with different $r/R$ ratios ($F = 100k \, \text{N}$, $r/R = 0.8$ and $t/R = 1.08$)
It is also found that the variation rate (slope) of the actuator driving force rapidly increased when the $r/R$ ratio decreased. These results are true for all the three links.

6. Again, $\phi = -60^\circ$ to $60^\circ$ and $\varphi = 45^\circ$ are used to show the effects of various $L/R$ ratios on the variation in the actuator driving forces of the machine tool. Figures 17, 18, 19 and 20 show the effects of $L/R$ on the driving axis variation with external cutting forces $F_{\text{ext}} = 100i_0 \text{N}$, $F_{\text{ext}} = 100j_0 \text{N}$, $F_{\text{ext}} = 100k_0 \text{N}$ and $M_{O_x} = 10000k_7 \text{N mm}$ respectively. By comparing Figs 17 to 20, no significant variation in the actuator driving forces is found when $L/R$ is varied ($L/R = 2$, $3$ and $4$) within the workspace range $\phi = \pm 40^\circ$. In general, the variation is smaller than 20 per cent.

6 SUMMARIES

The D–H notation method is used to investigate the effects of the structure design parameters on the force
results of a PKM tool. The singular location of the parallel-link machine is further obtained. A hybrid TRR–XY five-DOF machine tool is selected and investigated in this research. There are several results which are briefly summarized as follows:

1. From the forces of the joints or the singular position viewpoint, using a smaller cutter orientation in the machining process is preferred. For example, the cutter orientation of the hybrid TRR–XY PKM tool is suggested to be located within the range $\phi = \pm 50^\circ$ based on the structure design parameters.
2. The joint forces are significantly increased when the $r/R$ ratio is decreased. The workspace is slightly decreased when $r/R$ ratio is decreased.
3. The workspace is significantly increased when the $L/R$ ratio is increased. There is only a small joint force increase with increasing $L/R$ ratio.

![Fig. 18 Variation in actuator driving force with different L/R ratios (F = 100J N, r/R = 0.8 and t/R = 1.08)](image)

![Fig. 19 Variation in actuator driving force with different L/R ratios (F = 100k N, r/R = 0.8 and t/R = 1.08)](image)
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