Kinematics and workspace analysis of a six-degree-of-freedom optic fibre positioning stage using the Denavit–Hartenberg notation method

Fu-Shou Wang and Shang-Liang Chen*
Institute of Manufacturing Engineering, National Cheng Kung University, Taiwan, Republic of China

The manuscript was received on 24 April 2005 and was accepted after revision for publication on 2 December 2005.

DOI: 10.1243/09544054JEM369

Abstract: This paper analyses a hybrid optic fibre alignment positioning stage with six degrees of freedom (6-DOF) based on a 3-DOF parallel kinematic mechanism and a 3-DOF serial mechanism. The parallel kinematic mechanism uses three different types of ball joint to provide rotation and sliding movement. The advantages of the parallel kinematic mechanism include a simple structure, simplified kinematic analysis, zero accumulation of link errors, and high positioning accuracy. This study commences by introducing the parallel kinematic mechanism of the positioning stage and then proceeds to discuss its degrees of freedom. The Denavit–Hartenberg notation method is employed to analyse the kinematics of the positioning stage. Some simulation results are given to verify the correctness of this study and to show some potential for the optical fibre alignment applications. In the sequel, the effects of the dimensions of the mechanical components of the positioning stage on the workspace are also investigated and discussed.

Keywords: optic fibre alignment, positioning stage, parallel kinematic mechanism, workspace analysis, D–H notation method

1 INTRODUCTION

Using the Denavit–Hartenberg notation (D–H notation) method, the present study analyses the 6-DOF hybrid optic fibre positioning stage manufactured by Polamar, USA as shown in Fig. 1. This positioning stage incorporates a novel parallel kinematic mechanism comprising three different types of ball joint. The study commences by introducing the various kinematic mechanisms of the positioning stage and discussing the degrees of freedom of the parallel kinematic mechanism. A kinematic analysis of the positioning stage is then conducted and a homogeneous transformation matrix employed to obtain an inverse solution. This study concludes by discussing the effects on the workspace of the dimensions of the various mechanical components of the positioning stage.

Parallel mechanisms have been studied intensively for over a decade. The concept of a parallel kinematic mechanism evolved from the 6-DOF platform proposed by Stewart [1]. In the recent years, several novel parallel kinematic mechanisms have been conceived and designed and many interesting applications announced, including in simulators [2], industrial robots [3], parallel machine tools [4], and micromanipulators [5]. Compared to conventional mechanism, parallel kinematic mechanism provides manufacturers with a number of advantages, such as a higher loading capacity, higher structural stiffness, improved positioning accuracy, and a lower accumulation of manufacturing errors. However, these advantages come at the expense of a reduced workspace, complex mechanical design, and involved kinematics and control algorithms.

Several studies [6, 7] investigated parallel kinematic mechanisms that used ball joint engagement methods. The ball joints are used also in these studies to provide three-degrees-of-freedom (3-DOF) rotational movement. Lee [8] used geometric vector definition and Euler angle transformation to solve
the relationship between the kinematics and the geometry for parallel mechanisms. Wang [9] obtained the solid workspace of a parallel kinematic mechanism by evaluating discrete points on the plain to determine whether or not they satisfied the constraints of the mechanism. Chang [10] proposed a principle for optimizing the dimensional design parameters of a parallel kinematic machine tool. The three-dimensional workspace domain constructed by three parameters was divided into many points. These points were calculated using the algorithm of inverse solution, and it was judged whether they satisfy the constraints imposed by mechanism. The above-mentioned theories will be employed in this study for understanding the 6-DOF hybrid optical fibre positioning stage.

2 POSITIONING STAGE CONFIGURATION

Figure 1 presents a schematic illustration of the 6-DOF hybrid optical fibre positioning stage studied in the current research. The positioning stage comprises a novel 3-DOF parallel kinematic mechanism and a 3-DOF serial mechanism. The parallel kinematic mechanism differs from its conventional counterparts in that it incorporates three separate ball joints, each characterized by a particular joint type. The ball joints provide the following advantages: a simpler structure, straightforward up-and-down moving, measuring by optical meter, simplified movement formula, and zero accumulation of link errors. Although conventional parallel kinematic mechanisms are characterized by small workspaces, combining the parallel kinematic mechanism with a serial mechanism as in the present positioning stage provides a straightforward means of overcoming this limitation.

As shown in Fig. 1, the positioning stage comprises a base frame, a z axis stage, and an x axis stage. The yaw-axis rotation mechanism and the parallel kinematic mechanism are illustrated in Fig. 2. It can be seen that Link A, Link B, and Link C are oriented parallel with the y axis and that the end of each link is semispherical in shape. As part of this study, three springs were embedded between the work platform and the parallel kinematic mechanism to ensure a tight joint between the semispherical heads and the work platform.

As illustrated in Fig. 3, the semispherical heads and work platform are joined using three different types of joint, denoted as Joint A, Joint B, and Joint C respectively. Joint A is a semispherical head joined with a plane, Joint B is a semispherical head joined with a ball cave, and Joint C is a semispherical head joined with a slot formed between two semicylindrical columns.

The current parallel kinematic mechanism has a number of distinct characteristics, namely:

(a) it contains three sets of links positioned at 90°;
(b) the work platform cannot rotate along the y0 axis;
(c) Joint A and Joint C can not only rotate, but can also slide along the direction of the edge of the parallel kinematic mechanism;
(d) as the work platform inclination, the spinning axis must be parallel to the edge of the parallel kinematic mechanism;
the ball joints do not have the problem of accuracy caused by gap and can therefore deliver a high positioning accuracy. Hence, the kinematics characteristic can be simplified through the mechanism constraint.

3 ANALYSIS OF DEGREES OF FREEDOM

Analysing the degrees of freedom of the parallel kinematic mechanism, it is observed that Joint A has three rotational DOFs and two translational DOFs, Joint B has three rotational DOFs, and Joint C has three rotational DOFs and one translational DOF. From the formula for the degrees of freedom, the DOF $= 6(n_l - 1) - n_C = 6 \times (5 - 1) - [3 \times 5 + 1 + 2 + 3] = 3$, where $n_l$ is the number of members in the mechanism, and $n_C$ is the number of common constraints of the mechanism. Therefore, the current parallel kinematic mechanism has three DOFs, namely displacement in the y axis direction, rotation around the x axis, and rotation around the z axis.

4 D–H NOTATION METHOD FOR INVERSE KINEMATIC ANALYSIS

The D–H method [11] defines the homogeneous transformation matrix between two coordinate systems. The present study adopts this notation to solve the inverse kinematic problem of the current 3-DOF parallel kinematic mechanism, and sets the following parameters: link length $a$, offset $d$, twist angle $\alpha$, joint angle $\theta$. The D–H notation can be written as follows.
where \( i^{-1}A_i \) represents the D–H matrix from the \( i-1 \)th coordinate system to the \( i \)th coordinate system. Equation (1) enables the transformation matrix of the coordinate system for each kinematic chain to be obtained. As illustrated in Fig. 4, in the present case, the edge of the base is defined as the coordinate system \((XYZ)_0\). The coordinate system of each member can then be defined according to the \( B \) kinematic chain, and the centre position of the optic fibre defined as coordinate system \((XYZ)_t\). The coordinate transformation parameters for each kinematic chain are listed in Table 1. Note that the coordinate systems for links [6] to [8] are defined at the centre of the ball joint because each ball joint has a total of three different rotational degrees of freedom. Hence, three coordinate systems must be used to define the relation of movement.

The transformation matrix from coordinate system 0 to coordinate system \( t \) can be written as

\[
\left( ^0A_1 \right)_B = \left( ^0A_1 \right)_B \left( ^1A_2 \right)_B \left( ^2A_3 \right)_B \left( ^3A_4 \right)_B \left( ^4A_5 \right)_B \left( ^5A_6 \right)_B \left( ^6A_7 \right)_B \left( ^7A_8 \right)_B \left( ^8A_9 \right)_B \left( ^9A_{10} \right)_B \left( ^{10}A_{11} \right)_B \left( ^{11}A_{12} \right)_B \left( ^{12}A_{13} \right)_B \left( ^{13}A_1 \right)_B
\]

(2)

This matrix can be rearranged by combining the matrices containing no variables with the matrices containing some variables. The overall transformation matrix can then be written as

\[
\left( ^0A_1 \right)_B = \left( ^0A_1 \right)_B \left( ^1A_2 \right)_B \left( ^2A_3 \right)_B \left( ^3A_4 \right)_B \left( ^4A_5 \right)_B \left( ^5A_6 \right)_B \left( ^6A_7 \right)_B \left( ^7A_8 \right)_B \left( ^8A_9 \right)_B \left( ^9A_{10} \right)_B \left( ^{10}A_{11} \right)_B \left( ^{11}A_{12} \right)_B \left( ^{12}A_{13} \right)_B \left( ^{13}A_1 \right)_B
\]

\[
\begin{align*}
\left( ^0A_1 \right)_B &= \begin{bmatrix} 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} -\cos \theta_1 & \sin \theta_1 & 0 & 0 \\ -\sin \theta_1 & \cos \theta_1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
&\times \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
&\times \begin{bmatrix} -\cos \theta_2 & \sin \theta_2 & 0 & 0 \\ -\sin \theta_2 & -\cos \theta_2 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
&\times \begin{bmatrix} -\cos \theta_3 & \sin \theta_3 & 0 & 0 \\ -\sin \theta_3 & -\cos \theta_3 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\end{align*}
\]

Fig. 4 Schematic diagram of positioning stage coordinate systems

\[
i^{-1}A_i = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\]

(1)
### Table 1 D–H coordinate transformation parameters

<table>
<thead>
<tr>
<th>Link number</th>
<th>Kinematic chain A</th>
<th>Kinematic chain B</th>
<th>Kinematic chain C</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \theta )</td>
<td>( d )</td>
<td>( a )</td>
</tr>
<tr>
<td>1</td>
<td>-90°</td>
<td>(-a_1)</td>
<td>90°</td>
</tr>
<tr>
<td>2</td>
<td>-90°</td>
<td>(-a_2)</td>
<td>90°</td>
</tr>
<tr>
<td>3</td>
<td>180 + ( \theta_4 )</td>
<td>( d_3 )</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>( \theta_4 )</td>
<td>( d_{4A} )</td>
<td>-( a_4 )</td>
</tr>
<tr>
<td>5</td>
<td>( \theta_5 - \theta_4 )</td>
<td>( d_5 )</td>
<td>( a_5 )</td>
</tr>
<tr>
<td>6</td>
<td>180 - ( \theta_5 )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>( \theta_6 )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>90°</td>
<td>( \theta_7 )</td>
<td>0</td>
</tr>
<tr>
<td>9</td>
<td>( \theta_8 )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>( \theta_{10} )</td>
<td>( d_{10A} )</td>
<td>0</td>
</tr>
<tr>
<td>11</td>
<td>180 + ( \theta_{11} )</td>
<td>( d_{11} )</td>
<td>0</td>
</tr>
<tr>
<td>12</td>
<td>0</td>
<td>B</td>
<td>C</td>
</tr>
<tr>
<td>13</td>
<td>0</td>
<td>A</td>
<td>0</td>
</tr>
<tr>
<td>( t )</td>
<td>180</td>
<td>( t )</td>
<td>0</td>
</tr>
</tbody>
</table>

Additionally, \( P_{WL} = xi_W + yj_W + zk_W \) represents the distance between the origin of \((XYZ)_L\) and the origin of \((XYZ)_W\). For fibre alignment, the normal vector of the local coordinates \((XYZ)_L\) must be the same as the orientation of the fibre coordinates \((XYZ)_W\). In other words, the coordinate transformation matrix \( A \) must be equal to \( I_{6x4} \).

The transformation matrix of the base coordinate system \((XYZ)_0\) to the optic fibre centre coordinate system \((XYZ)\), via the positioning stage should equal the transformation matrix through the scan surface to the optic fibre centre coordinate system \((XYZ)_W\), i.e. equation (3) should equal equation (4). Therefore, the rotation angle and feedrate of each axis are given by

\[
\theta_3 = \tan^{-1}(i/k) \\
\theta_{6B} = \sin^{-1}(j) \\
d_1 = -a_{10}\sin\theta_{10}\cos\theta_{6B} + a_{10}\sin\theta_{10} + (A + i)\cos\theta_{6B}\times\cos\theta_3 \\
- (a_{10}\cos\theta_{10}\cos\theta_{6B} - d_{11}\sin\theta_{6B} + a_{10}\cos\theta_{10} + C\cos\theta_{6B} - B\sin\theta_{6B})\times\sin\theta_3 + \frac{a_2 - z + F}{\theta_3} \\
d_2 = -a_{10}\sin\theta_{10}\cos\theta_{6B} + a_{10}\sin\theta_{10} + (A + i)\cos\theta_{6B} + C\cos\theta_{6B} - B\sin\theta_{6B} \times \sin\theta_3 + \frac{a_2 - z + D}{\theta_3} \\
- a_{10}\sin\theta_{10}\cos\theta_{6B} - a_{10}\sin\theta_{10} + (A + i)\cos\theta_{6B} - d_{11}\sin\theta_{6B} - d_5 - a_3 - C\sin\theta_{6B} + (A + i)\sin\theta_{6B} - B\cos\theta_{6B} + y + E \tag{11}
\]

Note that the details of the corresponding derivations for the A and C kinematic chains are omitted in this paper because their inverse

\[
\mathbf{A}_W = \begin{bmatrix} i_W & j_W & k_W \end{bmatrix} \\
0 & 0 & 1 \\
0 & 0 & 0 \end{bmatrix} \cdot 4
\]

where \( k_W = hi_W + jf_W + kk_W \) is the unit vector of the normal direction of the scan surface.
solutions are the same as those described above. However, Joints A and C not only have rotational DOFs, but also have a translational DOF along the direction parallel to the edge of the work platform. Hence, as shown in Fig. 3, the distance $a_{10A}$ from Joint A to the centre of the work platform and the angle $\theta_{10A}$ are variable as the work platform moves. Similarly, the distance $a_{10C}$ and the angle $\theta_{10C}$ of Joint C are variable as the work platform moves.

Consequently, the relative functions of the A kinematic chain, B kinematic chain, and C kinematic chain can be obtained as

$$\theta_{dA} = \theta_{dB} = \theta_{dC} = \alpha$$

$$\theta_{SA} = \theta_{SB} = \theta_{SC} = \gamma$$

$$a_{10A} \sin \theta_{10A} = a_{10} \sin \theta_{10} \left( \frac{2 - \cos \theta_{6B}}{\cos \theta_{6B}} \right)$$

$$a_{10A} \cos \theta_{10A} = a_{10} \cos \theta_{10}$$

$$d_{4A} - d_{4B} = 2a_{10} \sin \theta_{10} \tan \theta_{6B}$$

$$a_{10} \sin \theta_{10} = a_{10C} \sin \theta_{10C}$$

$$a_{10C} \cos \theta_{10C} = a_{10} \cos \theta_{10} \left( \frac{2 - \cos \theta_{6B}}{\cos \theta_{6B}} \right)$$

$$d_{4B} - d_{4C} = 2a_{10} \cos \theta_{10} \tan \theta_{6B}$$

Equations (12) to (19) yield the feedrates, i.e. $d_{4A}$ and $d_{4C}$, the angles, i.e. $\alpha$ and $\gamma$, and the distance between the Joints A and C respectively and the work platform centre, i.e. $a_{10A}$ and $a_{10C}$.

### 5 WORKSPACE ANALYSIS

The workspace size is an important index of the performance of a positioning stage using parallel kinematic mechanism. The lower structure in the current positioning stage is a serial mechanism. A larger $X$ table, $Z$ table, and yaw-axis rotational angle will clearly yield a larger workspace. Hence, the workspace effects of the $X$ table, $Z$ table, and yaw-axis rotational angle need not be discussed in this study, i.e. attention can be focused solely on the parallel kinematic mechanism. Since the size of the workspace is limited by the parameters of the inclination angles $\alpha$ and $\gamma$, and the displacement in the $y$-direction, this study adopts two rotational DOFs and one translational DOF to construct a three-dimensional space. The three axes of this three-dimensional space are the inclination angle $\alpha$, the inclination angle, and the translation in the $y$-direction respectively. The workspace is solved by combining the inverse solution with the concept described in reference [10]. The three-dimensional workspace is constructed from the $\alpha, \gamma$, and $y$ parameters of many points. These points are calculated using the inverse solution algorithm, and each point is evaluated to determine whether or not it satisfies the constraints imposed by the mechanism. A Boolean function (0 or 1) is used to represent conformance to the constraint conditions. The points are set to 1 if they satisfy the constraints; otherwise they are set to 0, i.e. to indicate positions which the mechanism cannot reach. As shown in Fig. 5, the boundary of the
workspace is indicated by the transition of the Boolean function from 1 to 0.

In obtaining a high-precision positioning, the accuracy of the actuator is crucial. However, in the case of high precision, the actuator cannot have a larger stroke. Hence, this study adopts the feed stroke of the three links of the parallel mechanism as the constraint of the mechanism. The workspace size is related to the size of the mechanism. The major size parameter in the current mechanism is the centre distances between Joints A and B and Joints B and C respectively.

6 RESULTS AND DISCUSSION

Since the current optic fibre adopts an array arrangement, when performing the alignment scan of the optic fibre, it is necessary to carry out a surface scanning operation. As shown in Fig. 6, in surface scanning, a larger stepping size is employed initially to perform a rough scan and a local larger optic signal is obtained. A fine scan using a smaller stepping size is then performed in this local region to locate the maximum optic signal point. This point is then fixed and the yaw axis,
roll axis, and pitch axis are scanned in sequence to obtain the optimal alignment position and orientation.

Various scan rules are applicable to surface scanning, including line scan, box scan, and spiral scan, etc. This study adopts some of these rules to perform simulations, and sets the following machine parameters:

\[ a_1 = 22 \text{ mm}, \quad a_2 = 10 \text{ mm}, \quad d_3 = 14 \text{ mm}, \quad d_5 = 3 \text{ mm}, \quad OA_O = 52 \text{ mm}, \quad OB OC = 50 \text{ mm}. \]

The size of the fixture and mounting location are \( A = 59 \text{ mm}, B = 9 \text{ mm}, C = 0 \text{ mm}. \) The origin of the working coordinate system (\( XYZ)_{W}\) is situated at the base coordinate point (0, 0, 0). Based on these specifications, a spherical surface can be simulated in order to obtain the feedrate of each axis and the inclination angles of the work platform. The simulated spherical surface is presented in Fig. 7(a). The feed of X table has larger variation than the feed of Z table because the scanning points have orientation change. From Fig. 7(c), it can be seen that the feedrates of the links of the parallel mechanism are very smooth and are close to sine functions.

The simulation results for the spiral scan of the surface are shown in Fig. 8. This figure presents the orientation of the scan points vertical to the plane,
Fig. 9 (a) Simulation results for box scan path (step size of rough scan = 0.01 mm and step size of finish scan = 0.001 mm) (mechanism parameters: $a_1 = 22$ mm, $a_2 = 10$ mm, $d_3 = 14$ mm, $d_5 = 3$ mm, $O_AO_B = 52$ mm, and $O_BO_C = 50$ mm); (b) simulation results for feed of Z table; (c) simulation results for feed of X table; (d) simulation results for feed of Link A; (e) simulation results for feed of Link B; (f) simulation results for feed of Link C
i.e. the work platform has no tilting phenomena, and the feeds of the three links of the parallel mechanism are equal. When a variation of the scan points in the \( y \) axis direction occurs, the three links move up or down in a synchronized manner. To stimulate the actual scanning process, a 0.01 mm stepping size was used initially to conduct a rough box scan. A smaller 0.001 mm stepping size was then specified to carry out a fine box scan operation in order to establish the maximum optic signal point. Subsequently, yaw-axis, pitch-axis, and roll-axis orientation scanning operations were performed. The feedrates of the various axes are shown in Fig. 9. From this figure, it is clear that the feedrate of the serial mechanism varies only slightly in the \( x \) and \( z \) axis directions while surface scanning is performed. However, in the case of orientation scanning, the feedrate varies dramatically. Hence, it is necessary to the serial mechanism to have large stroke size for the orientation scanning. In achieving high-precision positioning, the accuracy of the actuator is crucial. However, the actuator with high precision is unable to have larger stroke. Hence, in the present study, the maximum stroke of the actuator is specified as 10 mm. In addition, the most important dimensional parameter of the parallel mechanism is the spacing of the ball joints. In the present simulations, these are set as \( O_A O_B = 52 \) mm and \( O_B O_C = 50 \) mm respectively. Initially, the \( y \) position is fixed, and the two-dimensional workspace boundary can be calculated. The two-dimensional workspace is then accumulated layer-by-layer to obtain a three-dimensional workspace, as illustrated in Fig. 10. From this figure, it can be seen that close to

---

**Fig. 10** Workspace constructed by \( \alpha \) and \( \gamma \) with various \( y \) axis positions: (a) three-dimensional wire frame model; (b) top view at \( Y = 5 \) mm; and (c) three-dimensional solid model

---

**Fig. 11** Workspace with different parameters: \( O_A O_B = 52 \) mm; \( Y = 5 \) mm; and actuator stroke of 10 mm
both sides of the $y$ direction is allowed only a unidirectional inclination of $\alpha$ or $\gamma$. Therefore, it is desirable to set the initial scanning point at the actuator half-stroke position in order to enable proper attention to both the $\alpha$ and $\gamma$ direction inclined angle while the orientation scanning is proceeding.

Figures 11 and 12 show the effect on the workspace of changing the centre distance of the ball joints. From these figures it can be seen that if $O_AO_B$ is fixed, the rotational angle of the work platform increases as $O_BO_C$ becomes smaller. However, the increase in $\alpha$ is greater than the increase in $\gamma$. Similarly, if $O_AO_C$ is fixed, the rotational angle of the work platform increases as $O_BO_A$ becomes smaller. However, in this case, the increase in $\gamma$ is greater than the increase in $\alpha$. Figure 13 illustrates the effect on the workspace of changing the height of the work platform. When $y=5$ mm, the rotational angle attains its maximum value. When the value of $y$ decreases, the workspace shifts toward the upper-left-hand side. Conversely, the workspace shifts toward the lower-right-hand side as the value of $y$ increases. Hence, an improved orientation scanning is achieved by commencing from the mid-stroke position of the actuator.

7 CONCLUSIONS

Using the D–H notation method, the kinematics and workspace of a hybrid 6-DOF optic fibre positioning stage are discussed in this research. From the present analysis it can be concluded that the areas of the $\alpha$ and $\gamma$ planes can be maximized by setting the initial scanning point at the mid-stroke position of the actuator. Regarding the influence of the mechanism parameters, it has been shown that a smaller distance between the two ball joints provides a greater inclination angle of the work platform. The present results enable a user to select mechanisms of appropriate dimensions to match the particular working requirements.

ACKNOWLEDGEMENTS

Parts of the research results were supported by the NSC of R.O.C (NSC 93-2212-E-006-106). The financial support is gratefully acknowledged.

REFERENCES


APPENDIX

Notation

\[ a_{10A}, a_{10}, a_{10C} \]

distance from ball joint to centre of work platform

\[ i^{-1}A_i \]

D–H matrix from \( i \) – 1th to \( i \)th coordinate system

\[ d_1, d_2 \]

feedrate of \( Z \) table and \( X \) table

\[ d_{4A}, d_{4B}, d_{4C} \]

feedrate of \( A \), \( B \), and \( C \) kinematic chains

\[ \alpha, \gamma \]

inclination angle of work platform

\[ \theta_3 \]

rotational angle of parallel kinematic mechanism