計畫類別：個別型計畫
計畫編號：
執行期間：公年 公月 公日至 公年 公月 公日
執行單位：國立成功大學物理學系（所）

計畫主持人：許祖斌

報告類型：簡報報告
報告附件：出席國際會議研究心得報告及發表論文
處理方式：本計畫可公開查詢
The research studies the constrinats equations and exact solutions of 4-dimensional canonical quantum gravity and their physical properties.
三、結果與討論

I summarize and briefly discussed the major results below:

**Result 1:**
We are able to prove that the super-Hamiltonian constraint of 4-dimensional gravity as simplified by Ashtekar through his use of gauge and triad variables can indeed be expressed succinctly as the Poisson Bracket of the Chern-Simons operator and the volume element operator. This, I judge, is a tantalizing and significant advance in the complete description of all non-perturbative states. The reason is that the quantum constraint equation (the Ashtekar-Wheeler-DeWitt Equation) now simplifies to the fact that the commutator of the Chern-Simons and volume element operator should vanish when acting on physical states. This translates into the fact that all states are eigenstates of these two operators. This can now be used as the guiding principle to work out the complete spectrum for all states.

**Result 2:**
What happens when a non-zero cosmological constant is present? Using previous experience from Ref. [1, 2, 3], our calculations yield a surprising bonus: the super-Hamiltonian constraints can still be expressed as the commutator of the Chern-Simons and volume element operator with the sum of the Chern-Simons operator and $F$ where $F = \int \text{Tr} (e \wedge D)$ is the generating function discussed in Ref. [3]. Thus the quantum constraint equation (the Ashtekar-Wheeler-DeWitt Equation) is still equivalent to the statement that all states are eigenstates of the volume element operator and the operator which is the sum of the Chern-Simons and $F$; and that their eigenstates provide may provide a complete solution to all the non-perturbative states.

**Result 3:**
The (regularized) volume operator emerged as most important, since all states in both results 1&2 are eigenstates of the volume element. In spin network states its spectrum is known to be discrete. Coupled with our observations above, this provides support that universe exists in states which have discrete volumes.

**Result 4:**
We are able to show that another fundamental quantity in general relativity, the extrinsic curvature, is also the commutator between $F$ and the gauge potential; and that the volume element operator acts as a “raising operator” for $F$. The first observation can help us to express the extrinsic curvature in terms of well-defined regularized integrated operator $F$ and the fundamental gauge potential variable, while the latter observation helps us understand the action of $F$ on physical states.

四、計畫成果自評

In my research proposal I judged this research project is broad in scope; requires careful, long and tedious calculations and study; and had applied for a two-year rather than a one-year National Science Council proposal.

I am happy to report the results obtained and discussed briefly above exceeded my own expectations, and I am in the process of writing up the results[4] for publication and will complete the major calculations this summer 2005. I have also been invited as a short-term visitor to the distinguished Perimeter Institute for Theoretical Physics, Waterloo, Canada, in the first-half of July 2005, during which I shall report on these results and discuss their significance with leading researchers in the area.

References:

Janeiro, Brazil 2003).

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