Growth of kidney and antikidney vortices over a square jet in crossflow

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The present paper reports on triple-decked evolution of the kidney vortices and the growth of unsteady antikidney-type vortices over a square jet in crossflow. Incompressible direct numerical simulations are performed with Reynolds number 2000 and jet to crossflow velocity ratio 2.5, to extract such flow details. Out of the three different decks, the lower deck kidney vortices maintain their steady appearance over the jet hole; however, vortices in the middle and the upper decks were unsteady in nature. Frequent shedding of the frontal hovering vortices and their convex wrapping over the front part of the jet hole are found to be responsible for the multidecked growth of the kidney vortices. On the other hand, unsteady antikidney-type vortices are noted to grow over the central part of the hole, and their growth is verified to be attributed due to concave windward folding of the frontal jet shear layer. Notably, the present findings remained consistent with existing experimentally observed features of such flows. © 2006 American Institute of Physics.

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Evolution of a jet in crossflow is quite a fundamental and frequently encountered flow phenomenon, and its practical applications cover the vast array of engineering/environmental problems. Upon emergence from the hole, as the jet quickly bends in the direction of crossflow, the vorticity both in the issuing jet shear-layer and the oncoming near-orifice boundary layer stretches and aligns to produce a multitude of dominant vortical structures, namely, the leading edge Kelvin-Helmholtz and hovering vortices, lee-side vortices, horseshoe-type vortices, and the counter-rotating vortex pair (CVP). Such experimentally measured features of the flow associated with a round jet are widely reported in Refs. 1–6. Furthermore, in recent years, attempts have been made to improve the cooling/mixing efficiency of a jet in crossflow by suitably introducing noncircular jets, such as those with square, rectangular, and elliptical geometry (e.g., Haven and Kurosaka7, New et al.8, Sau et al.9, Plesniak and Cusano10).

Notably, as the issuing front jet shear layer interacts with the oncoming crossflow, the hovering vortex is known to develop around the neck of the jet orifice. With the increase in Reynolds number (Re) and the velocity ratio (R) the hovering vortices are noted to shed periodically, and the quick lift up and realignment of such shedded vortices in the presence of the crossflow lead to the formation of multidecked nascent streamwise vortex structures (e.g., Haven and Kurosaka7, New et al.8) over the jet hole. Depending on the shape of the jet hole and the magnitude of the velocity ratio, the nature of the evolution of these near-hole streamwise vortices is noted to vary considerably, and they grow in the form of single, double, or triple-deck kidney/antikidney shaped vortical structures. For example, the streamwise vortices over a circular hole is noted to exhibit a single layer kidney shaped evolution pattern (as mentioned in New et al.8). On the other hand, according to New et al.8, depending on the aspect ratio, over an elliptic jet-hole such structures often evolve in single (for high aspect ratio), double, and triple-deck (low aspect ratio jet hole) kidney shaped vortices. Along the windward side the jet shear layer develops additional folds, leading to the generation of unsteady antikidney-type vortices. Furthermore, as Haven and Kurosaka7 report (and classify in p. 38 of their paper), over a turbulent square jet the near-field streamwise vortices evolve in the form of double-deck kidney shaped structures with unsteady upper deck appearing intermittently over the steady lower deck; and clearly there observed no trace of antikidney-type vortices (over a square jet hole). On the other hand, at a relatively lower Reynolds number, Re=225, and with velocity ratio $R=2.5$, Sau et al.9 noted single decked growth of steady kidney-type streamwise vortices over a square jet (presumably due to the fact that at such a low Reynolds number there occurred no periodic shedding of the hovering vortices). Naturally, questions arise about circumstantial validity and exhaustivity of such possibilities. It also remained to be investigated whether higher-decked (triple-decked) growth of the kidney vortices, or the growth of unsteady antikidney-type vortices are possible over a square hole as well, as they are noted to grow over an elliptical jet hole. On the other hand, while experimental findings of Haven and Kurosaka7 and New et al.8 express a similar view concerning the growth of kidney vortices, on the issue of the mechanism of the formation of antikidney vortices their findings are noted to deviate considerably. Objectives of the present investigation are, therefore, to try to significantly enhance our understanding concerning various possible phases of growth of the streamwise vortices over a square jet.
orifice, and determine the exact physical mechanism of formation for each of the above-mentioned kidney/antikidney-type vortices.

The flow field being investigated here is three-dimensional, and the upstream inlet section of the channel is placed at \(x = -4D\) (\(D\) being the jet width), through which fully developed crossflow fluid enters into the channel. A jet in the form of a square sectioned pipe flow then issues perpendicularly into the rectangular crossflow channel through an orifice which is placed symmetrically with respect to the channel span. With respect to the center of the jet orifice as the origin, the crossflow is spread in the \(x\) direction and the jet issues in the \(z\) direction with its axis coinciding with the \(z\) axis (see Fig. 1 in Ref. 9, for example). The exit section of the crossflow channel is located at a distance \(17D\) downstream of the jet orifice. Note that, the past experimental evidences clearly suggest that the proper growth of boundary layers both on the jet walls and on the crossflow channel floor is quite essential in the formation of the hovering vortex, and such issues have clearly been raised in Kelso et al.\(^4\)

In the present study we therefore consider fully developed jet flow through a pipe of length \(D\) and sectional area \(D \times D\), respectively. The lengths of the cross-flow channel along \(y\) and \(z\) axes are taken as \(7D\) and \(9D\), respectively.

The governing equations solved here are the unsteady incompressible Navier-Stokes equations in three dimensions, where all velocities are nondimensionalized by the mean cross-flow inlet velocity \(U_{cr}\), and the spatial coordinates have all been nondimensionalized by the jet width \(D\). Thus the Reynolds number is defined as \(Re = U_c D / \nu\). Another parameter that significantly influences the flow characteristics is the velocity ratio, \(R\), between the jet and the crossflow, and is defined as \(R = W_{jet} / U_{cr}\), where \(W_{jet}\) is the mean inlet velocity of the jet. In the simulation, fully developed axial velocity profiles were implemented at the inlet sections of the jet and the crossflow channel. The no-slip condition was implemented both on the jet boundary and on the crossflow channel floor, whereas, on the roof and on the side walls of the crossflow channel free-slip boundary condition was specified. At the downstream exit convective outflow boundary condition was implemented. The computational procedure employed here is an updated/modified version of the well known Marker-and-Cell method of Harlow and Welch.\(^{11}\) As far as numerical schemes are concerned, in brief, the convective terms in the momentum equations are discretized using a third-order accurate upwind scheme, and a fourth-order accurate central difference scheme is used to discretize the viscous terms. While over the years the numerical procedure has been widely used by many investigators, notably, in some of our recent works (e.g., Sau,\(^{12-14}\) Chiang et al.,\(^{15}\) Sau et al.\(^{9,16}\)) we carefully implemented the algorithm to study various complex flow phenomena associated with jet flows, and provided a detailed description of the simulation method.

In order to reveal the consistency of the present findings with the existing literature, in Fig. 1, we first depict various important symmetry plane features of the flow. Notably, the figure clearly exhibits, the growth of a horseshoe vortex system upstream of the jet hole, the folding of upstream and downstream interfaces of the jet due to developed Kelvin-Helmholtz-type instability, and the development of a node downstream of the jet hole. Fluid particles issuing from the symmetry-plane node are noted to move upstream (to join the jet), downstream, and vertically upward. In addition, the growth of several near-wall vortical structures and their eruption in layered form are some of the distinguishable features of the flow (e.g., Fric and Roshko,\(^3\) Kelso et al.\(^4\)).

One important aspect to note from Fig. 1 is the evolution of the leading edge hovering vortex located just above the jet hole. At a higher Reynolds number/velocity ratio this hovering vortex behaves in a transient fashion and often sheds gets lifted away from the orifice and moves downstream under the influence of the crossflow. In Fig. 1 we capture the hovering vortex as it remained spread around the hole. As the detached hovering vortex moves away from the orifice, the bottom-most Kelvin-Helmholtz vortex gets modulated due to sudden loss of stability and moves closer to the orifice. Under these conditions the large-scale roll-up of the jet shear layer appeared nearly periodic and occurred very close to the jet hole.

Past experimental observations suggest that the shedded frontal vortices, as they move upward with the jet, contribute to the multidecked evolution of the near-hole streamwise vortex structure. First, in order to demonstrate such a complex flow evolution process, in Fig. 2 we present streamwise vorticity contours at different instants of time. Notably, Fig. 2 clearly reveals that on a vertical plane \((x = -0.4)\), just behind the front edge \((x = -0.5)\) of the jet hole, the streamwise vortices often evolve through a triple deck structure, and sometimes they exhibit a double-decked evolution pattern. Moreover, all the three/two decks of vortices maintain the same sense of orientation as the kidney shaped CVP vortices (as mentioned in Haven and Kurosaka\(^2\) and New et al.\(^8\)). The formation of the double deck structure at the same Reynolds number (and velocity ratio) became possible, as the shedded upper deck vortices moved away from the plane \(x = -0.4\). Out of three different decks, the lower deck vortices maintain

![Image](http://pof.aip.org/pof/fig/pof10200197.jpg)
their steady appearance, however, vortices both in the middle and the upper decks exhibit unsteady characteristics. On the other hand, it is important to mention here that for a turbulent square jet (with \( R = 1.6 \)) Haven and Kurosaka\(^7\) noted the double-decked evolution of the streamwise vortices over the jet hole, and apparently they did not observe the presence of the third deck. However, for a low aspect-ratio elliptic jet hole, and apparently they did not observe the presence of double-decked evolution of the streamwise vortices over the contours ated at the cores of the three layers of kidney-type vorticity.

Interestingly, on \( x = 0.4 \) fluid particles constituting each of the three decks of vortices first turn toward left/right in a convex fashion (by taking downward turn presumably over the front jet interface), and in the process they contribute to the formation of three layers of kidney-type vortex pairs on \( x = 0.4 \) as depicted in Fig. 2(i).

Notably, as far as the source of the unsteady upper deck kidney vortices is concerned, both Haven and Kurosaka\(^7\) and New et al.\(^8\) mention that they (upper deck vortices) are the manifestation of convex wrapping of the shedded leading edge vortices passing periodically over their laser sheet. In order to explore/demonstrate such a complex flow evolution process, in Fig. 3 we draw stream-traces passing through the cores of each of the three layers of vortices as depicted in Fig. 2(i), and follow their path in three dimensions. For the sake of clarity, in Fig. 3 we also depict six small stencils \( L_1, L_2, M_1, M_2, U_1, U_2 \) (on the plane \( x = 0.4 \)), physically situated at the cores of the three layers of kidney-type vorticity contours [as indicated by arrows in Fig. 2(i)], through which streamlines penetrate. Notably, the streamline behavior in Fig. 3 clearly displays that on the plane \( x = 0.4 \) fluid particles constituting each of the three decks of vortices first turn toward left/right in a convex fashion (by taking downward turn presumably over the front jet interface), and in the process they contribute to the formation of three layers of kidney-type vortex pairs on \( x = 0.4 \) as depicted in Fig. 2(i).

Therefore, as far as the mechanism of growth of the kidney vortices is concerned, Fig. 3 clearly displays the consistency of the present findings with those of Haven and Kurosaka\(^7\) and New et al.\(^8\). It may also be noted from Fig. 3 that during streamwise evolution, the fluid particles issued from all three decks experience an upward (concave) fold at a downstream location. However, a careful examination of the flow (by placing a number of vertical planes over the jet hole, and observing from different angles) reveals that the second (concave) fold of the streamlines occurred at a relatively far downstream location, and therefore the same vortex element was possibly not responsible for the growth of the antikidney vortices (that are observed to form over the central part of the orifice).

As far as antikidney-type vortices are concerned, notably, Haven and Kurosaka\(^7\) did not observe their presence over a square jet hole. However, both Haven and Kurosaka\(^7\) and New et al.\(^8\) did observe unsteady antikidney-type vortices to grow over high-aspect-ratio elliptic jet holes. Furthermore, on the issue of physical mechanism of formation of the antikidney vortices, findings of Haven and Kurosaka\(^7\) and New et al.\(^8\) deviate considerably. We now therefore explore the feasibility of growth of antikidney-type vortices over a square jet hole, and investigate their physical process of generation. Our understanding is that, it is the contours of \( \sqrt{u^2 + w^2} \) which can most closely demonstrate cross-stream (normal to crossflow) flow evolution pattern in a way similar to those presented in the form of LIF images in New et al.\(^8\). Interestingly, on \( x = 0.15 \) the depicted contours in Fig. 4 clearly reveal the antikidney shaped local evolution of the jet column (with \( R = 2.5 \)), and such a trend was noted to continue until \( x = 0.2 \). Notably, behind the jet orifice, all such locally grown kidney and antikidney-type vortices were eventually entrained into the CVP core [Fig. 2(iii)]. It is important to mention here that, with the increase of velocity ratio the growth of primary steady CVP vortices (as termed in New et al.\(^8\)) became more and more clear on \( x = 0.3 \), however, limitation of space restricts us from presenting such results in greater detail. Here we may point out that, the

![FIG. 2. Multidecked temporal growth of streamwise (\( \omega_z \)) vorticity on a vertical plane (\( x = 0.4 \)) normal to the crossflow. Re=2000, \( R = 2.5 \).](image1)

![FIG. 3. (Color online) Stream-traces passing through the cores of triple-decked \( \omega_z \) vortices demonstrate the involved physical process of convex wrapping of the shedded frontal vortex sheets on \( x = 0.4 \) leading to the growth of kidney shape vortical structures on the plane. Re=2000, \( R = 2.5 \), \( t = 17 \).](image2)
contours in Fig. 4 represent the 2D projected view of a flow which is truly three-dimensional. Therefore, in order to shed further light into the mechanism of local flow evolution, in Fig. 5 we depict 3D stream-traces passing through the topmost parts of the antikidney shaped structures, as presented in Fig. 4(i). For the sake of clarity, small sections of the plane $x=-0.15$ (the stencils $A_L$ and $A_R$) through which respective stream-traces penetrate have also been depicted in Fig. 5. It may be noted that upon approaching $x=-0.15$ (the plane $ABCD$) the streamlines in Fig. 5 experience clear concave (upward) windward wrapping, which influences antikidney shaped local evolution of the jet structure. In order to be sure about the nature of local folding of the frontal jet shear layer, we plotted such 3D pictures at different instants of time, and it appears that Figs. 5 and 3 best reveal the physical mechanisms leading to the development of antikidney and kidney vortices, respectively. Therefore, it is quite likely that the source of antikidney vortices is the windward concave wrapping of the leading edge vortex sheet. Notably, the unsteady kidney/antikidney-type vortices, observed in the present study, are of the same nature as the primary unsteady vortices, as classified in New et al.\(^8\) In summary, the present investigation reveals that the kidney vortices over a square jet can in fact evolve in a triple-deck form (not necessarily in double deck form as reported in Haven and Kurosaka\(^7\)), and there also takes place the growth of unsteady antikidney-type vortical structures over the central part of the jet hole (which Haven and Kurosaka\(^7\) apparently did not observe). While the growth of the kidney vortices (over the front edge of the jet hole) is caused by the convex wrapping of the leading edge vortex sheet, a concave windward folding of the deflected frontal jet shear layer over the central part of the jet orifice facilitates the growth of the antikidney-type vortices.

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FIG. 5. (Color online) Stream-traces passing through the topmost parts of the antikidney shaped structures indicating the process of concave windward wrapping of the deflected frontal vortex sheets on $x=-0.15$. Re=2000, $R=2.5$, $t=17$. 

FIG. 4. Contours of $\sqrt{\overline{u'^2}+\overline{w'^2}}$ revealing the growth of antikidney-type unsteady near-hole vortical structures on a plane normal to the crossflow. Re =2000, $R=2.5$. 