Intrinsic instability and locking of pulsation frequencies in free-running two-mode class-B lasers

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We report on recent experimental results of the dynamics of a laser-diode-pumped free-running Nd:YVO₄ laser operating in a two-mode regime. We observe intrinsic quasiperiodic and chaotic oscillations as well as a locking of pulsation frequencies. We perform an asymptotic analysis of model rate equations in which an intensity-dependent cross-gain (i.e., nonlinear gain) mechanism of direct mode-mode coupling is introduced in addition to the coupling mechanism via cross saturation of population inversions. We show that the intrinsic instability originates from the nonlinear gain mechanism. The observed locking of pulsation frequencies is successfully reproduced by simulations based on the proposed rate equations.

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I. INTRODUCTION

Free-running multimode class-B lasers utilizing Fabry-Perot cavities have been most successfully described by Tang, Statz, and deMars (TSD) rate equations [1]. Both a linear [2,3] and nonlinear [4] analysis of the TSD equations for an arbitrary number of lasing modes give rise to self-organized dynamics such as antiphase but not to any kind of unstable steady states.

Among many kinds of solid-state laser materials belonging to class-B, widely used Nd:YVO₄ (yttrium orthovanadate) laser crystals, which are of significant current technological interest as diode-pumped microchip lasers, possess unique spectroscopic properties such as reduced Stark splitting and substantial transfer of lattice energy to Nd ions [5]. These features result from an unusual crystal field which is strongly modified by lattice vibrations (e.g., phonons) [5,6]. Consequently, the YVO₄ host causes substantially large inhomogeneous broadening [6]. In addition, Raman spectroscopic studies showed that microinhomogeneities depend on the method of crystal growth [7].

Recently, quasiperiodic and chaotic evolutions as well as a $p/q$-type locking of modal pulsation frequencies (i.e., intermode parametric resonance [8,9]) have been observed in a laser-diode (LD)-pumped free-running Nd:YVO₄ laser operating in a two-mode regime without any additional external influence factors [10]. The $p/q$-type locking implies that the first lasing mode exhibits period-$q$ pulsations and the second lasing mode does period-$p$ pulsations, in which the repetition periods of the two modes are the same. Self-induced spiking oscillations and associated instabilities have also been demonstrated using the same laser free running in a three-mode regime [11]. These experimental observations shed light on the fundamental understanding of laser stability since it is commonly believed that spontaneous instabilities are impossible in free-running class-B lasers without any additional degree of freedom such as external modulation, light injection, intracavity second-harmonic generation, delayed feedback, etc. Various physical mechanisms have been thus incorporated to the TSD rate equations with the hope of predicting unstable steady solutions. Those additional mechanisms explored up to date are phase-sensitive coupling between the complex modal amplitudes [12–14] via the long-wavelength population grating [12,15,16], population diffusion [17], and longitudinal inhomogeneous pumping [15,16]. However, none of the above listed mechanisms suits the conditions of our experiments [10,11]. Since in the Nd:YVO₄ laser the phonons of the host lattice strongly modify the crystal field, this results in a third-order nonlinearity. Such an intensity-dependent nonlinearity is thought to induce a cross-gain mechanism of direct mode-mode coupling which could play a key role in triggering spontaneous instabilities.

In this paper we present detailed analytical and numerical studies of the dynamics of a two-mode free-running laser-diode-pumped microchip Nd:YVO₄ laser together with experimental results. The experimental setup and observations for the two-mode regime are presented in Sec. II. Section III is devoted to an asymptotic analysis of model rate equations with a higher-order cross-gain mode-coupling term which was proposed to describe a class of inhomogeneously broadened lasers like that under consideration. The analysis clarifies the origin of the instability occurrence. Finally, we show the simulation results in Sec. IV. The instability threshold and the observed phenomenon of $p/q$-type locking of pulsation frequencies are well reproduced by our simulations.

II. THE EXPERIMENT

An experimental setup is shown in Fig. 1. Experiments were carried out by using a LD-pumped Nd:YVO₄ laser. The laser crystal (CASIX) was 1-mm thick and dielectric mirrors were coated on both ends of the crystal. A collimated lasing beam from the laser diode oscillating at $\lambda_p=808$ nm (Opto-
Power Corp.; 1 W) was passed through anamorphic prism pairs to make an elliptical beam circular and was focused onto the laser crystal by a microscope objective lens (M 20×). A linearly π-polarized TEM_{00} mode along the crystal c axis was observed in the entire pump domain. The lasing threshold was \( P_{\text{th}} = 26.5 \text{ mW} \) and the slope efficiency was 30%. The lasing spectrum was changed by slightly changing the pump position since lasing mode frequency arrangement changes within the asymmetric gain profile with a 0.96-nm bandwidth [full width at half maximum (FWHM)] with a slight change in the crystal (i.e., cavity) thickness where the longitudinal mode spacing was 0.246 nm for the 1-mm cavity length. A single longitudinal-mode oscillation at \( \lambda = 1064.5 \text{ nm} \) was obtained below \( P = 51 \text{ mW} \). Above this value the laser exhibited two longitudinal-mode oscillations where the second mode appeared at \( \lambda = 104.25 \text{ nm} \). Above \( P = 210 \text{ mW} \), three longitudinal-mode oscillations appeared. The modal output was detected by an InGaAs photodiode (New Focus 1811; 125-MHz bandwidth, 3-ns rise time) after passing it through a spectrometer with a 0.01-nm resolution. The intensity waveform was measured by a digital oscilloscope (Tektronix TSD 420A) and the power spectrum was measured by a rf spectrum analyzer (Tektronix 2712). The input-output characteristics, observed locking steps of modal pulsation frequencies and a chaos region are summarized in Fig. 2. In the region of \( p/q \)-type locking shown in Fig. 2, the first lasing mode showed period-q pulsations while the second lasing mode exhibited period-p pulsations in which the repetition periods of the two modes were the same. In this example, the 2/5-type locking was observed in the pump region 2.6 < \( w < 3.2 \) and the 1/2-type locking was attained in the wide pump region \( w > 3.9 \), where \( w = P/P_{\text{th}} \) is the relative pump power. Examples of observed power spectra of the total output for 2/5-type locking and 1/2-type locking behaviors are displayed in Figs. 3(a) and 3(b), respectively. This type of locking behavior is similar to intermode parametric resonance phenomenon reported in multimode lasers subjected to simultaneous modulations at multiple rational frequencies with \( f_2/f_1 = p/q \) ratio near relaxation frequencies [8], in which the total output power spectrum exhibits peaks with \( p/q \) frequency ratio. Such lockings are persistent features over a range of pump power indicated in Fig. 2 rather than occurring at a specific pump power. In other pump regions, quasiperiodic pulsations were observed, in which \( p \) and \( q \) are incommensurate numbers, and the spectrum is broadened in region C with a sudden transition to chaos [10].

### III. The Analysis

To clarify the origin of observed intrinsic instability we describe the free-running multimode Fabry-Perot class-B Nd:YVO_{4} laser by the model rate equations proposed in [10]. These equations are an extended version of the TSD rate equations in which a cross-gain mechanism due to the intensity-dependent nonlinearity that directly couples the modes is introduced in addition to the well-known cross-saturation mechanism due to spatial hole burning that indirectly couples the modes via population gratings. For the case of the two-mode regime the model rate equations are of the form

\[
\frac{\partial D_1}{\partial t} = w - D_1 - D_1(I_1 + \beta I_2),
\]

(1)

\[
\frac{\partial D_2}{\partial t} = w - D_2 - \gamma D_2(I_2 + \beta I_1),
\]

(2)
\[ \frac{\partial I_1}{\partial t} = \kappa \{ [(1 - \eta I_2)D_1 - 1]I_1 + \alpha D_1 \}, \] (3)

\[ \frac{\partial I_2}{\partial t} = \kappa \{ [(1 - \eta I_1)D_2 - 1]I_2 + \alpha D_2 \}. \] (4)

In Eqs. (1)–(4) \( I_n (D_n) \) is the normalized modal intensity (population inversion), \( \kappa = \tau_f / \tau_p \) with \( \tau_f \) the fluorescence lifetime and \( \tau_p \) the cavity photon lifetime, \( t \) is the time measured in units of \( \tau_f, \gamma (\gamma < 1) \) is the ratio between the linear gains of mode 2 and mode 1 with mode 1 the first lasing mode, \( \alpha \) is the spontaneous-emission coefficient, and \( w \) is the homogeneous pump scaled to the threshold of the first lasing mode. The indirect coupling among modes due to spatial hole-burning is described by \( \beta \). The direct mode-mode coupling due to a crossgain mechanism is described by \( \eta \). We remind the reader that the introduction of the direct mode-mode coupling mediated by the parameter \( \eta \) is motivated by the third-order nonlinearity present in Nd:YVO\(_4\) lasers due to phonon-induced modifications of the crystal field. Note that in practice the spontaneous-emission coefficient is very small, of the order of \( 10^{-7} \), and would give negligible corrections for the whole laser dynamics. We can therefore set \( \alpha = 0 \) in our analysis.

Let \( \{ \bar{I}_1, \bar{I}_2, \bar{D}_1, \bar{D}_2 \} \) be the steady-state solution of Eqs. (1)–(4). These equations admit four types of steady-state solutions. The nonlasing solution,

\[ \bar{I}_1 = \bar{I}_2 = 0, \quad \bar{D}_1 = \bar{D}_2 = w, \]

exists for any \( w \) but is stable for \( w < 1 \) only. There are two kinds of single-mode solutions. One kind is determined by

\[ \bar{I}_1 = 0, \quad \bar{I}_2 = w - 1, \quad \bar{D}_1 = \frac{\gamma w}{\gamma - \beta + \beta \gamma w}, \quad \bar{D}_2 = \frac{1}{\gamma}, \]

which exists for \( w > 1 / \gamma \) and is stable for

\[ w > w_{12} = \frac{1}{2 \gamma \eta} \left[ \gamma - \beta \gamma + \eta \right] \]

\[ \pm \sqrt{(\gamma - \beta \gamma + \eta)^2 - 4 \gamma \eta (\gamma - \beta)}. \]

The other kind of single-mode solution is determined by

\[ \bar{I}_1 = w - 1, \quad \bar{I}_2 = 0, \quad \bar{D}_1 = 1, \quad \bar{D}_2 = \frac{w}{1 + \beta \gamma (w - 1)}, \]

which exists for \( w > 1 \) and is stable for \( w < w_{21} \) and \( w > w_{22} \) where

\[ w_{21,22} = \frac{1}{2 \gamma \eta} \left[ \gamma (1 - \beta + \eta) \right] \]

\[ \pm \sqrt{\gamma^2 (1 - \beta + \eta)^2 - 4 \gamma \eta (1 - \beta \gamma)}. \]

Finally, the two-mode solution is found to be

\[ \bar{I}_1 = \frac{\gamma \eta w^2 - (\gamma - \beta \gamma + \eta)w + \gamma - \beta}{\gamma (\beta + \eta w)^2 - 1}, \]

\[ \bar{I}_2 = \frac{\gamma \eta w^2 - \gamma (1 - \beta + \eta)w + 1 - \beta \gamma}{\gamma (\beta + \eta w)^2 - 1}, \]

\[ \bar{D}_1 = \frac{1}{1 - \eta I_2}, \]

\[ \bar{D}_2 = \frac{1}{\gamma (1 - \eta I_1)}. \]

For given \( \gamma, \beta, \) and \( \eta \) the two-mode solution (9)–(12) exists in discrete intervals of the pump determined by

\[ w_{21} < w < w_{22} \quad \text{and} \quad w > w_{12}, \]

where \( w_{12}, w_{21}, \) and \( w_{22} \) are defined by Eqs. (6) and (8). On the other hand, for given \( w, \beta, \) and \( \eta \) the two-mode solution emerges for

\[ \gamma > \gamma_{\text{min}} = \frac{1}{\beta + (1 - \beta + \eta)w - \eta w^2}. \]

We now probe stability of the two-mode solution by an asymptotic analysis based on \( \kappa \gg 1 \). This proves to be a very good approximation since our interest is on microchip Nd:YVO\(_4\) lasers or/and similar solid-state lasers for which \( \tau_f \gg \tau_p \) (typically, \( \kappa = \tau_f / \tau_p \approx 10^3 - 10^6 \gg 1 \)). Because the effect of nonlinear gain is practically small, we assume \( \eta = B / \kappa \ll 1 \). Then, as a result of linear stability analysis, the time-dependent perturbation of the two-mode solution is proportional to a linear superposition of \( \exp(\lambda_j t) \) with

\[ \lambda_j = (-1)^j \sqrt{\kappa \omega_j} + \Gamma_j + O(1 / \sqrt{\kappa}), \quad j = 1,2,3,4. \]

In Eq. (15)

\[ \omega_{1,2} = \sqrt{\bar{I}_1 + \bar{I}_2 - \Delta}, \quad \omega_{3,4} = \sqrt{\bar{I}_1 + \bar{I}_2 + \Delta}, \]

\[ \Gamma_{1,2} = -\frac{1}{4 \sqrt{\Delta}} \{ w[(1 + \gamma) \sqrt{\Delta} - (1 - \gamma)(\bar{I}_1 - \gamma \bar{I}_2)] \]

\[ - 2 B \beta (1 + \gamma) \bar{I}_1 \bar{I}_2 \}, \]

\[ \Gamma_{3,4} = -\frac{1}{4 \sqrt{\Delta}} \{ w[(1 + \gamma) \sqrt{\Delta} + (1 - \gamma)(\bar{I}_1 - \gamma \bar{I}_2)] \]

\[ + 2 B \beta (1 + \gamma) \bar{I}_1 \bar{I}_2 \}, \]

where

\[ \Delta = (\bar{I}_1 + \gamma \bar{I}_2)^2 - 4 \gamma (1 - \beta^2) \bar{I}_1 \bar{I}_2 = (\bar{I}_1 - \gamma \bar{I}_2)^2 + 4 \gamma B^2 \bar{I}_1 \bar{I}_2 \]

and
From Eq. (16) it is evident that \( \Gamma_{1,2}^{(0)} < 0 \) since \( \gamma < 1 \) and, by virtue of Eq. (19),

\[
\bar{\Gamma}_1 - \bar{\Gamma}_2 = \frac{(1 - \gamma)(\beta(1 + \gamma) + w(1 - \beta))}{\gamma(1 - \beta^2)} > 0.
\]

It can be verified that \( \Gamma_{1,2}^{(0)} \) is also negative when Eq. (18) is used for \( \Delta \) in Eq. (20). Hence, the two-mode solution is perfectly stable without cross-gain coupling.

When the cross-gain coupling is taken into account, from Eqs. (16) and (17) it follows that \( \Gamma_{3,4} \) remains negative but \( \Gamma_{1,2} \) may become positive if the coupling strength \( B \) satisfies the inequality

\[
G \mathbf{w}(1 - \beta) - \mathbf{w} = \Gamma_{1,2}^{(0)} - \frac{w(1 - \beta)}{\gamma(1 - \beta^2)}. \tag{19}
\]

Note that in the above formulas \( \bar{\Gamma}_{1,2} \) are nothing else but the steady-state modal intensities without the cross-gain coupling, i.e., for \( \eta = 0 \).

If the cross-gain coupling is neglected then Eqs. (16) and (17) reduce to

\[
\Gamma_{1,2}^{(0)} = -\frac{w}{4 \sqrt{\Delta}}[(1 + \gamma) \sqrt{\Delta} - (1 - \gamma)(\bar{T}_1 - \gamma \bar{T}_2)], \tag{20}
\]

\[
\Gamma_{3,4} = -\frac{w}{4 \sqrt{\Delta}}[(1 + \gamma) \sqrt{\Delta} + (1 - \gamma)(\bar{T}_1 - \gamma \bar{T}_2)]. \tag{21}
\]

From Eq. (21), it is evident that \( \Gamma_{3,4}^{(0)} < 0 \) since \( \gamma < 1 \) and, by virtue of Eq. (19),

\[
\bar{T}_1 - \gamma \bar{T}_2 = \frac{(1 - \gamma)(\beta(1 + \gamma) + w(1 - \beta))}{\gamma(1 - \beta^2)} > 0.
\]

Figure 4 displays a phase diagram in the \((w-B)\) plane for the two-mode regime. The neutral stability curve \( B_c \) separates the stable from the unstable domain. Note that

\[
limit_{w \to \infty} B_c = \frac{(1 + \beta)[(1 + \gamma) \sqrt{(1 - \gamma)^2 + 4 \beta^2} - (1 - \gamma)]}{2 \beta(1 + \gamma)^2} \tag{22}
\]

\[
\gamma \to 1 + \beta. \tag{23}
\]

Figure 4 clearly shows that the stable two-mode domain is getting narrower and narrower for increasing \( B \). In our experiment the value of \( \kappa \) was on the order of \( 2.5 \times 10^6 \), in which \( \tau_g = 90 \) \( \mu s \), so that the corresponding values of \( B \) is large for reasonable values of \( \eta = 10^{-3} \sim 10^{-2} \). Therefore, the two-mode stationary regime occurs in a negligibly small range of pump values and was not observed experimentally. In fact, we have observed immediately after the second-mode threshold \( P = 51 \) mW chaotic emissions followed by domains of locked pulsation frequencies as summarized in Fig. 2. Figure 5 is another phase diagram in the \((\gamma-B)\) plane. In the limit of identical modes \( \gamma \to 1 \), we get

\[
limit_{\gamma \to 1} B_c = \frac{w(1 + \beta)}{w - 1}. \tag{24}
\]

From Figs. 4, 5, Eq. (23), and Eq. (24), the role played by the different coupling mechanisms is elucidated. Namely, the cross-gain coupling \( \eta \) tends to destabilize the laser operation while the cross-saturation coupling \( \beta \) tends to stabilize the laser operation. Likewise, the observed intrinsic instability is
explicitly explained as being originated from the proposed cross-gain coupling mechanism.

IV. THE SIMULATION

In this section we numerically integrate the proposed model rate equations to justify their validity and to confirm the analytical results obtained from them. For the two-mode regime the simulation with $k > 10^4$ agrees perfectly with the analytical results represented in Figs. 4 and 5. Namely, for parameters below the neutral stability curves $B_c$ damped oscillations are obtained while above $B_c$ quasiperiodic and chaotic oscillations appear. The phenomenon of $p/q$-locking of pulsation frequencies is also reproduced numerically in Figs. 6 and 7 for the two-mode regime. Here, we assumed $k = 5 \times 10^5$ which is smaller than the real value of $10^6$ in the experiment since such a large $k$-value resulted in a computer artifact, i.e., stiff problem. However, it should be noted that physics did not change depending on the $k$ values we used. In Fig. 6 it is clearly seen that the first lasing mode exhibits period-5-like behavior whereas the second lasing mode behaves like period 2. We also plot in Fig. 6 the total intensity power spectrum with sharp peaks at frequencies whose ratio is equal to $f_2/f_1 = 2/5$.

FIG. 6. Numerical confirmation of a 2/5-type of locking of pulsation frequencies. The simulated waveforms of two-mode pulsations with $w = 2.47$, $\kappa = 10^3$, $\gamma = 0.535$, $\beta = 1/15$, $\eta = 0.02$, and $\alpha = 1.2 \times 10^{-7}$. (a) The first lasing mode exhibits period-5-like behavior, (b) the second lasing mode exhibits period-2-like behavior, and (c) the total intensity power spectrum with sharp peaks at frequencies whose ratio is equal to $f_2/f_1 = 2/5$.

pump power, i.e., $w > 4$, the simulation results reproduce the observed 1/2-type of pulsation frequency locking. The calculated $w$ regions for these locking steps agree with the observed ones shown in Fig. 2. A typical manifestation of 1/2-locking is illustrated in Fig. 7 in which the first lasing mode exhibits period-2-like behavior, the second lasing mode behaves like period 1, and the total intensity power spectrum shows sharp peaks at frequencies whose ratio is equal to $f_2/f_1 = 1/2$. Furthermore, chaotic pulsations were also reproduced numerically just above the threshold of the second lasing mode similarly to the experimental result.

V. CONCLUSION

To sum up, we have investigated experimentally, analytically, and numerically the dynamics of a two-mode Fabry-Perot free-running laser-diode-pumped microchip Nd:YVO$_4$ laser. We have observed the occurrence of intrinsic instabilities in a free-running laser without any external influence factors. The origin of instability is analytically shown to be due to the direct mode-mode coupling by a cross-gain mechanism. This nonlinear gain mode-coupling arises in solid-state laser materials like Nd:YVO$_4$ in which the host lattice vibrates strongly, resulting in an unusual crystal field and thus giving rise to an additional possibility of direct coupling among lasing modes as compared to previously used materials such as a Nd:YAG (yttrium aluminum garnet).
frequencies is interesting and is also reproduced successfully by simulations based on our rate equations. Good agreements of the observed results with the analytical as well as the numerical results do justify the validity of the proposed model rate equations at least for the description of two-mode free running class-B lasers like Nd:YVO$_4$. It is hoped that these model rate equations remain adequate also in regimes of any number of lasing modes.

FIG. 7. Numerical confirmation of a 1/2-type of locking of pulsation frequencies. Same as in Fig. 6 except for $w = 6.5$. (a) The first lasing mode exhibits period-2-like behavior, (b) the second lasing mode exhibits period-1-like behavior, and (c) the total intensity power spectrum with sharp peaks at frequencies whose ratio is equal to $f_2/f_1 = 1/2$. Such a 1/2-locking effect is obtained for a wide range of $w$, say, from $w = 4$ up to $w = 8$. (For a larger value of $w$ a three-mode regime appeared in the experiment.)

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