Abstract

The present study uses Johnson's transformation with approximate test statistics to test the homogeneity of simple linear regression slopes when both $x_{i,j}$ and $\epsilon_{i,j}$ may have nonnormal distributions, and there is Type I heteroscedasticity, Type II heteroscedasticity, or complete heteroscedasticity. The idea is that the test statistic $t$ is first transformed by Johnson's method for each group to correct nonnormality, and then, to correct heteroscedasticity, that an approximate test, such as the Welch test or the Deshon-Alexander test, is applied for testing the homogeneity of the regression slopes. Computer simulation result shows that the proposed technique can control Type I error rate under various circumstances. Finally, a real example is provided to demonstrate the calculation.

Introduction

Violating assumptions of statistical test in the regression context is not widely recognized in the educational and psychological literature (Alexander & DeShon, 1994; Wilcox, 1997a, p.207). In fact, however, testing the equality of $J$ independent regression slopes when heteroscedastic error variances are present is urgently needed in educational and psychological research settings, including differential validity and prediction research, analysis of covariance (Huitema, 1980), or aptitude-treatment interaction (ATI). Moreover, nonnormal distribution is very common in applied work (Micceri, 1989), which leads to low statistical power. Wilcox (1987, 1998) also demonstrates assumption evaluation and therefore advocates robust methods. Consequently, the present study considers the simple linear regression model with the presence of nonnormality and Type I, Type II, or complete heteroscedasticity and proposes an
alternative to lessen the effect of outlying observation and heteroscedasticity. The present study considers two variance configurations: $v(x) = 1$ and $v(x) = 1 + 2/(1 + |x|)$. The former is referred to Type II heteroscedasticity or the homogeneity of the residual variances which is taken to mean that the variances differ between groups (Dretzke, Levin, & Serlin, 1982). And the latter is referred to Type I heteroscedasticity which corresponds to small error variances when the value of $x$ is in the tails of its distribution. Moreover, complete heteroscedasticity refers to a situation where both Type I and II heteroscedasticity conditions are present (Wilcox, 1997b).

To correct Type II heteroscedasticity, Deshon and Alexander (1996) adapted Alexander and Govern's (1994) approximate statistic to test the slope equality. However, they obtain incorrect Type I probabilities if the distribution is nonnormal or in the outlier case.

To correct nonnormality, on the other hand, Johnson (1978) modified the $t$ variable by using the first few terms of the inverse Cornish-Fisher expansion, and also by substituting sample estimates for the second and the third central moments contained in the expansion. His method is useful for distributions with skewness as severe as that of the exponential distribution. Sutton (1993) confirmed that Johnson's modified $t$ should be preferred to the classical $t$ test when the parent distribution is asymmetrical.

Considering the joint impact of nonnormality and heteroscedasticity, the present study proposes using Johnson's transformation in conjunction with the approximate tests (i.e., Deshon-Alexander test, 1996; Welch test, 1951) to improve the robust properties of previous methods. Moreover, the present study considers $x$ variable to be random in order to investigate the performance of different statistics under various assumption violations. Finally, a computer simulation experiment and a real example in comparing two curriculums are provided.

**Conclusion and Discussion**

It is well known that psychological and educational data are likely to violate the assumption of a normal distribution. This often occurs in combination with heterogeneous error variances. Hence, some regression analyses might encounter a degree of heteroscedasticity and nonnormal error distributions. The present study shows that the practical importance of the Cornish-Fisher Johnson's transformation in conjunction with the approximate methods can both be adopted. The present study examines the small-sample properties, via computer simulations, of the approximate transformation method.
for simple regression, and the results indicate that the proposed technique should be used when the number of regression lines increases and the distribution is skew with Type II heteroscedasticity. In addition to the Welch test and the A-G test, the present study also investigated the James second-order test (James, 1951) and found that its pattern of simulated Type I errors is similar to the other two tests. Although not shown here, the tables of those results are available from the first author. The computer program for calculating the proposed test statistics is also available on request.

Considering the problem of estimating the regression function underlying a set of data that is contaminated, continued work in this area is important, especially in the area of robust/resistant estimators (Wilcox, 1998). Trimmed mean, nonparametric methods, or new hybrid methods are another important direction for this research. Furthermore, application of the robust procedure has been limited almost exclusively to the case of a single explanatory variable with the response variable. The aspect of dimension is one additional regression problem. For the most part, regression with only one independent variable is well understood. But when we move to two, three, or even more independent variables, the situation changes. The robust version of any estimators should be generalized easily to higher dimensions.

REFERENCES


Communications of the ACM, 13, 619-620.


James, G. S. (1951). The comparison of several groups of observations when the ratio of the population variances are unknown. Biometrika, 38, 324-329.


