Our research topic was the synthesis and design of multi-band filters. We mainly focused on the computation of general multi-band filtering functions fitting some pre-defined specifications. One of our objectives was in particular to obtain filtering functions that can be used to design wideband multi-band filters.

This research led to two papers that were accepted to international conferences:
[2] Lunot, V.; Tsai, C.-M.; "Bandpass Computation of Asymmetric Multi-band Filtering Functions with Improved Rejection"; 2013 IEEE MTT-S International Microwave Symposium; Seattle, June 2013

The two above papers are attached to this work report.

In [2], an efficient way to compute asymmetric multi-band functions associated to a real admittance is presented. This type of function has the advantage that it could be used to design wideband multi-band filters. It is next possible to extract from them, thanks to classical techniques, a circuit using only stub elements. Some work still has to be done to convert this circuit to one more practical, i.e. with only unit elements in serie, and shunt open-circuit and short-circuit stubs.

Another topic that we studied is the generalization of the Butterworth filter to multi-band specifications. Indeed, for multi-band filters, all the current research is only focusing on the generalization of the Chebyshev or Elliptic filters. However, such filters as Butterworth or Bessel would also be of interest.
We discovered that in the case of multi-band filters, a Butterworth filter can in general be only exactly realized by using a characteristic function with complex coefficients. This means in particular that such a filter can only be exactly realized for narrowband specifications.
In Fig. 1, the normalized response of a filter of order 10 is plotted. Due to the strong constraints, the rejection between the bands is not strong.
In Fig. 2, the zeros of the filtering function of order 10 are plotted. We see that such zeros are not 'conjugated two by two' with respect to 1 and -1.
This work led us assume that the case of the generalization of the Bessel filter to multi-band specifications would have the same limitation. It would probably require a characteristic function with complex
coefficients to be exactly realized.

Fig. 1: response of a normalized dual-band Butterworth filter of order 10

Fig. 2: zeros of the dual-band Butterworth filtering function plotted in Fig. 1
二、研究或教學或科技研發與管理成效評估（由計畫主持人或單位主管填寫）

Please evaluate the performance of research, teaching or science and technology R&D and management work: (To be completed by Project Investigator or Head of Department/Center)

(1) 是否達到延攬預期目標？
Has the expected goal of recruitment been achieved?

Yes. Dr. Lunot has a lot initiative in the approximation problems. Part of his research has been written as two conference papers.

(2) 研究或教學或科技研發與管理的方法、專業知識及進度如何？
What are the methods, professional knowledge, and progress of the research, teaching, or R&D and management work?

Dr. Lunot is doing very well in his part of work. His approximation method has gradually joined with the network synthesis established by Electromagnetic Lab of NCKU. New directions of filter design have been discovered. The progress of this project is satisfactory.

(3) 受延攬人之研究或教學或科技研發與管理成果對該計畫(或貴單位)助益如何？
How have the research, teaching, or R&D and management results of the employed person given benefit to the project (or your unit)?

Dr. Lunot's work on approximation problem complements the research on the network synthesis of microwave multi-band filters by our Electromagnetics Lab. of EECS. This kind of joint research is very rare in the world. We have discovered new topics which cannot be found in any literatures.

(4) 受延攬人於補助期間對貴單位或國內相關學術科技領域助益如何？
How has the employed person, during his or her term of employment, benefited your unit or the relevant domestic academic field?

Dr. Lunot has helped to develop the mathematical theory in microwave filter synthesis. In Taiwan, there is very few (if any) research in this area. Even in the world this research is considered very special. The completed theory is expecting to be able to create new methods in microwave filter synthesis.

(5) 具體工作績效或研究或教學或科技研發與管理成果:
Please describe the specific work performance, or the results of research, teaching, or R&D and management work:

Dr. Lunot has sent out two different papers to two major symposiums in his research area. Both of them have been accepted. The second one also has a great chance to be extended to a full journal paper in IEEE transactions on Microwave Theory and Techniques.

(6) 是否續聘受聘人？Will you continue hiring the employed person?  □ 繼聘 Yes  ■ 不續聘 No

Dr. Lunot’s performance is good.
We would be more than happy to keep him here if we can locate the needed funding for this research.

※此報告表篇幅以三～四頁為原則。This report form should be limited to 3-4 pages in principle.
※此表格可上延攬優秀人才成果報告繳交說明網頁下載。This report form can be downloaded in http://scholar.lib.ncku.edu.tw/explain/
Bandpass Computation of Asymmetric Multi-band Filtering Functions with Improved Rejection

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Abstract—We present here an algorithm to compute asymmetric multi-band filtering functions in the bandpass domain. The result is a function that is symmetric in absolute value with respect to zero frequency but asymmetric with respect to the central frequency of the passbands. The algorithm leads to a response optimized in magnitude. One of the advantages is that the computed filtering function is not limited anymore by the lowpass to bandpass transformation. It is therefore possible to compute filtering functions with a better response than the one obtained by transformation from the lowpass circuit. A direct consequence is that by choosing different number of transmission zeros at zero frequency, more choices of circuits are also available. We illustrate the method by the study of an asymmetric dual-band filter of order seven.

Index Terms—Asymmetric response, Remez algorithm, multi-band filter, wideband filter.

I. INTRODUCTION

Microwave bandpass filters are usually first designed as lowpass circuits that are next transformed to bandpass circuits (e.g. [1]). This process has the advantage to simplify the computations and the overall design. However, the choice of the lowpass to bandpass transformation always lead to some limitations. Furthermore, the design of lowpass circuits with an asymmetric response requires the use of frequency-invariant reactive elements ([2]) and is therefore limited to narrowband filters. In [3], a method for the computation of single-band Chebyshev functions directly in the bandpass domain has been presented. The main advantage is that the function is not limited anymore by the lowpass to bandpass transformation and can be used to design filters with an arbitrary bandwidth.

We propose in this manuscript a method for the computation of multi-band filtering functions in the bandpass domain. The result is a function that is symmetric in absolute value with respect to zero frequency but asymmetric with respect to the central frequency of the passbands taken as a whole. Such a characteristic function always has real coefficients. We show on an example that by choosing different numbers of transmission zeros at zero frequency, the rejection between the passbands can be strongly modified. A set of numbers of zeros at zero frequency that lead to a response meeting the specifications may next be selected. Each number of zeros corresponding to circuits with different types of elements, the overall number of choices is increased.

The heart of the process is an algorithm of type Remez ([4]-[5]). Such type of algorithm has already been proposed for computation in the lowpass domain ([6]-[8]). We present below an improved algorithm that allows to compute a polynomial having some given specific constraints in magnitude and that is nonnegative on \( \mathbb{R}^+ \). A filtering function with real coefficients and same gain response can next be identified. A weight is introduced in the computation in order to handle different numbers of transmission zeros at zero frequency.

To illustrate the method, we study an all-pole asymmetric dual-band filter of order seven. The filter is realized thanks to Richards’ theory ([9]). The size of the bands are first modified in order to compensate the error due to the Richards’ transformation. Next, the algorithm is used to compute a characteristic function in the bandpass domain.

II. OVERVIEW OF THE METHOD

We denote by \( \chi = p/q \) the characteristic (or filtering) function (e.g. [1]). When considering \( p \) and \( q \) as polynomials with real coefficients in the complex plane, we notice that their absolute values on the pure imaginary axis are symmetric with respect to zero. Indeed, if \( p \) is a polynomial of degree \( n \) with real coefficients \( (a_k)_k \) then

\[
|p(j\omega)|^2 = \text{Re}[p(j\omega)]^2 + \text{Im}[p(j\omega)]^2
\]

\[
= \left( \sum_{k=0}^{\lfloor n/2 \rfloor} a_{2k}(-1)^k \omega^{2k} \right)^2 + \left( \sum_{k=0}^{\lfloor n/2 \rfloor} a_{2k+1}(-1)^k \omega^{2k+1} \right)^2.
\]

It is therefore straightforward that \( \omega \to |p(j\omega)|^2 \) is an even polynomial with real coefficients and nonnegative over \( \mathbb{R} \). This is also true for \( |q(j\omega)|^2 \) and consequently \( |\chi(j\omega)|^2 \) is symmetric with respect to zero frequency.

Furthermore, given an even polynomial \( P \) of degree \( 2n \) with real coefficients and nonnegative over \( \mathbb{R} \), we can always find a polynomial \( p \) of degree \( n \) with real coefficients such that \( |p(j\omega)|^2 = P(\omega) \) for all \( \omega \). Indeed, since \( P \) is nonnegative, all its roots have even multiplicity. Let's denote by \( r_k, 1 \leq k \leq m_1 \) the positive real roots of \( P \) taken with only half of their multiplicity. Since \( P \) is even, \((-r_k)_k \) are also roots of \( P \). As for the complex roots of \( P \), since \( P \) has real coefficients, they are conjugated two by two and their opposites are also roots because \( P \) is even. We pick up the complex roots with positive real part and positive imaginary part and we denote them by \( c_k, 1 \leq k \leq m_2 \). We denote by \( m_0 \) half of the multiplicity of the root zero. Then, if \( \gamma > 0 \) is the leading coefficient of \( P \),

\[
p(x) = \sqrt{\gamma} x^{m_0} \prod_{k=1}^{m_1} (x - jr_k)(x + jr_k) \prod_{k=1}^{m_2} (x - jc_k)(x + jc_k)
\]

is a real polynomial that satisfies \( |p(j\omega)|^2 = P(\omega) \) for all \( \omega \).
Given a nonzero polynomial \( Q(x) = x^d \), the method is therefore as follows:

1) We first compute a polynomial \( P \), real, nonnegative on \( \mathbb{R}^+ \) and such that \( P/Q \) has an optimal rejection.
2) We next identify two polynomials \( p \) and \( q \) such that
\[
|p(j\omega)|^2 = P(\omega^2) \quad \text{and} \quad |q(j\omega)|^2 = Q(\omega^2).
\]
The main algorithm used in step 1 is described in the following section.

III. REMEZ-TYPE ALGORITHM

To compute polynomials nonnegative on \( \mathbb{R}^+ \), we chose to extend the algorithm from [6]. Indeed, improving this algorithm has two main advantages compared to modifying the ones from [7] or [8]. First, it is the only algorithm that guarantees a maximal rejection. This is illustrated in Fig. 1: the bandpass transformation of the lowpass “Chebyshev” function of order 7, that has 4 reflection zeros in the lowpass band and 3 in the right one, has a better rejection outside of the left passband than the transformed “equiripple” function from [7] or [8] with same order and same number of zeros in each band. The second advantage is that extending the algorithm from [6] results in an algorithm that is computationally less expensive than the ones obtained by modifying [7] or [8]. Indeed, in [7] or [8], the number of reflection zeros in each passband has to be chosen before computation. Therefore, different numbers of zeros in each band have to be tried in order to find the best function. However, the algorithm used here doesn’t require any initial guess: the zeros are automatically distributed in order to get a maximal rejection.

In [6], the computed function is in absolute value bounded by 1 in the passbands and has a maximum rejection in the stopbands with respect to given signs. It can be checked that rather than considering functions being bounded between -1 and 1 in the passbands, the algorithm can also be used for a set of functions bounded between 0 and 1. Next, choosing all the signs positive, we see that if we are able to add a nonnegativity constraint between the bands, we obtain the expected algorithm.

Consider a set of passbands and stopbands disconnected two by two. We denote by \( I \) the union of all the passbands, \( J \) the union of the stopbands and by \( Q \) the absolute value of a nonzero polynomial. Let \( A \) be the set
\[
A = \left\{ p \in P_n, p(x) \geq 0 \forall x \in \mathbb{R}^+ \setminus J, \frac{p(x)}{Q(x)} \leq 1 \forall x \in J \right\}
\]
where \( P_n \) is the set of real polynomials of degree at most \( n \). The problem to solve is to find \( P \) such that
\[
P = \arg \max_{p \in A} \inf_{\omega} \frac{p}{Q}.
\]
The algorithm is similar in nature to the one in [6]. Given an initial sequence of \( n + 2 \) points (the choice only has an impact on the number of iterations needed to converge), the three following steps are repeated:

1) Compute the solution \( p_k \) of the problem over the set of \( n + 2 \) points \( x_1^k, \ldots, x_{n+2}^k \).
2) Look for the point \( y_k \) where the solution over \( n + 2 \) points is the farthest from the constraints.
3) Substitute \( y_k \) to one of the points in the set \( \{ x_j^k \}_{j \neq k} \) in order to get a new sequence of \( n + 2 \) points \( x_1^k, \ldots, x_{n+2}^k \).
It stops at step 2 when all the points are close enough to the constraints.
Note that compared to the computation in lowpass, the complexity is increased since the degree of the polynomials is doubled. However, no choice of signs is required anymore.

IV. EXAMPLE: A 7-POLES DUAL-BAND FILTER

We consider a dual-band filter with two passbands centered at 1.1GHz and 2.35GHz with respective widths of 5% and 4%. The two passbands are \( [1.072, 1.127] \) and \( [2.303, 2.397] \). The return loss is given at 20dB and the rejection has to be at least 50dB at 1.7GHz.
We want to realize this filter using Richards’ theory. The commensurate frequency is taken at 10GHz.

We first modify the passbands in order to compensate the narrow band approximation of the Richards’ transformation. Indeed, since we synthesize the two passbands as a whole, the difference between the frequency at the beginning of the first band and the frequency at the end of the second band is large. The widths of the passbands are therefore “distorted” by the transformation. The point where the commensurate lines will be equivalent to the LC elements is chosen at 2.09GHz. It is around that frequency that the absolute “distortion” is the less important. The computation will therefore be done for the two modified passbands \( [1.044, 1.098] \) and \( [2.322, 2.426] \).
For reference, the functions obtained from the classic lowpass to bandpass transformation are plotted on Fig. 1. We remind that since the filter is asymmetric with respect to its central frequency, these functions don’t have real coefficients. We can see that the filtering function of order 7 has a poor rejection between the passbands with a maximum close to 43dB. However, the filtering function of order 8 has a way stronger rejection between the bands with a maximum around 68dB. We therefore expect to need a filter of order 8 to reach our specifications.

We remind that the classic lowpass to bandpass transformation sends half of the transmission zeros to zero frequency and the remaining ones to infinity. By computing directly in the bandpass domain, we can choose the number of transmission zeros at zero frequency. In Fig. 2, we see that we found a filtering function of order seven with a rejection of almost 60dB between the bands. In comparison to the function obtained from the classic lowpass to bandpass transformation, we got an improvement of more than 16dB. Such an improvement is explained by two factors. First, due to the non-linearity of the classic lowpass to bandpass transformation and the distance between the bands, the respective sizes of the bands are distorted. Therefore, the optimal numbers of reflection zeros in each
passband when computed in lowpass is different from the ideal numbers in bandpass. But as we can see on Fig. 2, the choice of seven transmission zeros at zero frequency is also far from being optimal. For a filter of order 7, way better results are obtained for a small number of transmission zeros, the overall best response in rejection having only one. Note that we also computed the functions of order 8 and found that the one with the best rejection has 7 transmission zeros at zero frequency. The maximum of its rejection between the two passbands is close to 70dB. This better explains the strong difference of rejection in Fig. 1 between the functions of order 7 and 8. Whereas 7 transmission zeros at zero frequency is close to the worst case for the filter of order 7, this number is close to the best case for the filter of order 8.

We computed the admittance associated to the characteristic function of order 7 with one transmission zero at zero frequency. Using the classic Richards' theory, we obtained the values of the different elements of the bandpass circuit. The response of the ideal transmission lines circuit is given in Fig. 3. Thanks to the high commensurate frequency, the mirrored passbands due to the transformation are sent far away from the frequencies of interest.

V. CONCLUSION

An algorithm for the computation of multi-band filtering functions in the bandpass domain has been presented. In comparison to a classic transformation from the lowpass circuit, this algorithm allows in particular to choose different number of transmission zeros at zero frequency, and therefore to focus either on improving the rejection or providing a bigger choice of circuits to realize the filter. The only drawback is that the complexity of the computation is increased. The case of an all pole filter of order seven has been presented and synthesized thanks to the classic Richards' theory.

REFERENCES

Exact Synthesis of a Compact Dual-band Filter with an Asymmetric Response

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Abstract— Microstrip dual-band filters with commensurate transmission-line sections are usually designed based on the periodicity of the Richards' variable $j \tan \theta l$ [1]. The quarter-wavelength frequency is therefore between the two passbands. We propose here to directly synthesize a dual-band bandpass filter as a single-band filter whose band contains the two passbands. The quarter-wavelength frequency is therefore taken higher than the upper bound of the second passband and consequently all the transmission-line sections become shorter. Furthermore, when the periodicity of the Richards' variable is used in the traditional way, the second passband is a transformed version (mirror) of the first one, and therefore the number of zeros in each band is always the same. Synthesizing directly the two passbands as proposed allows to take a different number of zeros in each band. One problem of the method presented here is that when the two passbands are far from each other, realizing directly the filter is equivalent to designing a wideband filter, which is usually limited by the requirement of ideal coupling elements. We will consider such an example and therefore compute a filtering function that doesn't require any ideal coupling elements.

The example taken was designed for the wireless LAN applications. The central frequencies at the two passbands are 2.45 and 5.25 GHz, and the bandwidths are respectively 10% and 3%. The quarter-wavelength frequency is taken at 10 GHz. We consider an all poles filter of order 9 to realize these specifications. We first compute a lowpass response thanks to a Remes-like algorithm [2] that guarantees an optimal rejection. Note that such an optimal function is not equiripple and that it has two complex conjugated zeros in the first band. We next transform the lowpass response to the real frequencies thanks to the classic lowpass to bandpass transformation. Since the lowpass response is asymmetric, the bandpass response is also asymmetric with respect to the real frequency zero, and consequently doesn't lead to a reflection function $S_{11}$ with real coefficients. We therefore modify the bandpass response to obtain a reflection function with real coefficients by using only the right-hand plane zeros. The response of the modified function is close to the original one and both responses are plotted in Figure 1. Darlington's synthesis [3] is finally used to realize the filter.

![Figure 1: Comparison of the transformed low pass function (blue) and the obtained symmetric function (green) for (b) real frequencies and (a) "negative frequencies".](image-url)
REFERENCES